On Generalized Empirical Bayes Updating*

Thomas Augustin THOMAS.AUGUSTIN@STAT.UNI-MUENCHEN.DE
Foundations of Statistics and Their Applications, Department of Statistics, Ludwig-Maximilians-Universität München (LMU Munich)

Main Idea This contribution revisits the venerable question of updating imprecise probabilities. We briefly discuss several variants of a generalized empirical Bayes rule for learning from a sample $\boldsymbol{x}$ given a set $\mathscr{M}$ of prior probabilities. By applying Bayes rule only to a sample-based refinement/adaption $\mathscr{M}_{|\boldsymbol{x}}$ of $\mathscr{M}$, these rules provide a natural compromise between Walley's generalized Bayes rule and the so-called ML-II updating rule (initially suggested by Good, 1983, e.g., p. 46f), an empirical Bayes approach, which is also intimately related to Dempster's rule of conditioning. The different variants of the generalized empirical Bayes approach proposed here aim at posterior conclusions that are neither distorted by extremely unlikely prior probabilities nor by the overoptimism of the ML-II approach.

Operationalizations of the Main Idea Within the Bayesian machinery, a natural way to refine/adapt $\mathscr{M}$ is to ask how well its elements predicted the concretely observed sample $\boldsymbol{x}$. This means we draw on the marginal probabilities $(\boldsymbol{p}_\pi(\boldsymbol{x}))_{\pi(\cdot)\in\mathscr{M}}$ of the sample value $\boldsymbol{x}$, after having integrated out via the prior distribution π over its domain. Based on this, two directions to proceed will be considered.

Type-A models rely on an externally fixed threshold δ and consider all elements of $\mathscr{M}$ that have a relative predictive power of at least δ, setting $\mathscr{M}_{|\boldsymbol{x}} = \left\{ \pi \in \mathscr{M} \,\middle|\, \frac{\boldsymbol{p}_\pi(\boldsymbol{x})}{\max_{\pi\in\mathscr{M}} \boldsymbol{p}_\pi(\boldsymbol{x})} \geq \delta \right\}$. This approach has been introduced and studied by Cattaneo (2014), with a strong emphasis on the continuity properties of the updating process.

Type-B models are robustifications of the ML-II method. They look at the set $\widehat{\mathscr{S}_{|\boldsymbol{x}}}$ of priors maximizing $(\boldsymbol{p}_\pi(\boldsymbol{x}))$ and take a neighbourhood $\mathscr{U}(\widehat{\mathscr{S}_{|\boldsymbol{x}}})$ around it, possibly intersected with $\mathscr{M}$. A preliminary, pragmatically motivated application of a Type-B approach in small area estimation can already be found in Omar and Augustin (2018, p. 180f.).

Computational Aspects For finite parameter spaces, the calculations of Type-A and Type-B models can be transferred into fractional linear optimization problems. For the general case, closed forms of prior predictive distributions for parametrically constructed models exist (see also Quaeghebeur and de Cooman, 2005).

Next Steps Further research includes i) a more careful investigation of the relationship of our approach to congenial ideas from IP and robust Bayesianism, ii) deeper thoughts on the compatibility of our approach with models for proper handling of prior-data conflict, as well as iii) a critical appraisal of the potential of our approach for constructing hierarchical models for Bradley & Hill's (personal communication) confidence-based reasoning in climate impact modelling by varying δ in Type-A models or the neighbourhood radius in Type-B models.

References

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