

An Informational View of Belief Functions*

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Belief functions are a well-established formalism for the representation of uncertainty, with multiple applications and interpretations.

In set-theoretical terms, any belief function Bel over a certain frame of discernment Ω can be obtained, starting from a basic probability assignment m over the subsets of Ω . The degree of belief $Bel(A)$ of a subset A of Ω is then the sum of the probabilities $m(B)$, for all the subsets B of A .

This can be reformulated in logical terms, as *probability of provability* (see e.g. [4, 3, 2]), i.e. assuming that a probability assignment is given for arbitrary logical formulas and that the belief $Bel(A)$ in a formula A is obtained as the sum of the probability of the formulas B which allow to formally derive A in classical logic. Though clearly mathematically equivalent, the logical framework presents the advantage of isolating the inferential part in the formation of the degree of belief, which might be obscured in the purely set-theoretic rendering.

The model of inference which classical logic presents is, however heavily idealized, and in particular computationally unfeasible, according to the theoretical standards of computational complexity. We can hardly conceive of a realistic agent forming its degrees of belief in a proposition, in the way discussed above, by way of performing any, arbitrarily complex, classical inference.

We consider thus a more refined model of inference, based on the informational framework of *Depth-Bounded logics* [1], with the aim of providing more realistic models of belief. Depth-Bounded logics rely on a fundamental distinction between two kind of inferences: the ones that manipulate information actually possessed by an agent and the ones which require the manipulation of *virtual* information, i.e. information not actually present to the agent, but nevertheless relevant for logical inference. The former kind of inferences determine a logical consequence relation called 0-depth. Starting from such logic, a hierarchy of higher k -depth logics is defined, which allow the manipulation of virtual information at increasing complexity. All the k -depth logics are computationally feasible, and provide approximations of classical logic, being equal to the latter at the limit.

Correspondingly, in our work we introduce a hierarchy of *depth-bounded belief functions*, which are associated to k -depth consequence relations, and permit a more fine-grained distinction between the manipulation of virtual and actual information, when determining degrees of belief. We assume that an original probability assignment m_0 is given to the formulas at 0-depth, and that probability assignments m_k corresponding to higher depths are obtained by suitable conditioning. To each k -depth assignment m_k , a belief function B_k is then associated, in terms of the probability of k -depth provability.

We obtain then that any classical Belief function can be seen in our perspective as the limit of a suitable sequence of k -depth Belief functions B_k , in the same way as classical logic can be seen as the limit of the k -depth logics.

References

- [1] Marcello D'Agostino, Marcelo Finger, and Dov M. Gabbay. Semantics and proof-theory of depth bounded boolean logics. *Theor. Comput. Sci.*, 480:43–68, 2013. doi:[10.1016/j.tcs.2013.02.014](https://doi.org/10.1016/j.tcs.2013.02.014). URL <https://doi.org/10.1016/j.tcs.2013.02.014>.
- [2] J. B. Paris. *The Uncertain Reasoner's Companion: A Mathematical Perspective*. Cambridge University Press, 1994.
- [3] Judea Pearl. Reasoning with belief functions: An analysis of compatibility. *Int. J. Approx. Reasoning*, 4(5-6):363–389, 1990. doi:[10.1016/0888-613X\(90\)90013-R](https://doi.org/10.1016/0888-613X(90)90013-R). URL [https://doi.org/10.1016/0888-613X\(90\)90013-R](https://doi.org/10.1016/0888-613X(90)90013-R).
- [4] P. Smets. Probability of provability and belief functions. 34(133-134), 1991.

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