Social Pooling of Beliefs and Values With Desirability

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This work is concerned with the question of aggregating beliefs (probabilities) and values (utilities) of a given number of rational subjects. The problem is of a foundational and philosophical nature; at the same time it has concrete statistical implications given that in applications we often want to aggregate information coming from different sources, or even predictions of different models.

Not surprisingly, the problem has a long history in the literature. A prominent example is *social choice theory* [2], which aims at defining social functions that best represent the preferences of a group of rational voters. In this context, the celebrated *Arrow theorem* [1] establishes limits to what is rationally possible to do while avoiding dictatorial solutions; these limits are severe in particular with complete preferences. Social choice theory is concerned with preferences over simple options (such as candidates to an election). As such it is not directly concerned with questions of probability. The related research field of *probabilistic opinion pooling* is instead concerned with finding a model that best 'summarises' a given number of probabilistic beliefs. Interestingly, some recent research [4] argues that imprecise probability has much potential in opinion pooling, in that precise probabilistic approaches incur problems that remind the Arrowian limitations.

In this work we address the aggregation problem with the formalism of desirability. This has a few main advantages: the formalism is equivalent to that of preferences over horse lotteries and for this reason we can simultaneously deal with considerations of beliefs and values [6]; the framework is very general also because we can deal with any domain and possibility space [7]; moreover, it allows us to work in opinion pooling using preferences rather than probability, and this makes it much easier to carry over to pooling some of the ideas developed in social choice.

On this basis, we initially study how some of the traditional results [3, 5] in social choice transform in our setting: dictatorship, oligarchy, autocracy and democracy. Then we provide our main result: coherence essentially implies linear pooling, that is, the idea of aggregating preferences (i.e., beliefs and values) via convex mixtures. This seems to indicate that the aggregation problem can be solved in a principled way, in particular when we use imprecise probability.

References

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