

On the Dimensionality of Credal Networks with Interval Probabilities

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Statement: In this poster we state the following general property of credal networks:

The number of decision variables (or dimensionality) required to compute the inference in two-state credal networks with interval probabilities grows at most linearly with the number of nodes directly connected to the queried variable. We present a proof of this statement using rigorous interval analysis together with a numerical example. To demonstrate the applicability of the obtained property a general algorithm will also be presented.

Complexity of Inference: The number of arithmetical operations required by Bayesian inference in credal networks grows exponentially with the number of nodes that are directly connected to the queried variable. Bayesian inference consists in computing the posterior probability of the queried node conditional to its child's (or parent's) probability. For example, a two-state network with one node connected to N parent nodes has 2^N terms in the conditional probability table, and thus 2^N terms in the summation to obtain the posterior probability. When the network has interval probabilities the inference becomes an optimisation task where the posterior probability is the objective function, and each term in the conditional probability table is a decision variable, so there are as many as 2^N decision variables in this case. The optimisation aims at finding the lower and upper bounds on the posterior probability. Efficient implementation of inference algorithms for polytree-shaped credal networks can be found in [1] to mention an example.

The Bayesian inference problem in credal networks, is an optimisation with an exponentially growing number of decision variables, where the exponent is the number of nodes directly connected to the queried variable. The problem of interval propagation in credal networks has already been studied by Tessem in [3], who proposes an approximate method to compute bounds on the inference probability. This contribution only partially complements the already copious literature on the complexity of credal networks with findings, for example, on binary and ternary polytrees.

Solution: We show that the number of decision variables required to solve the interval problem of finding lower and upper bounds, can grow at most linearly, as opposed to exponentially, with the number of connected nodes. This is shown using the concept of *interval gradient* with which we can rigorously check the monotonicity of the posterior probability against the decision variables. An interval problem that is monotonic with respect to some decision variables, can be solved more efficiently, because all the bounded decision variables for which the problem is monotonic can be assigned to their endpoints with constant values [2]. The interval gradient, which is obtained using automatic differentiation, can be deployed to (i) classify which variables can be assigned, and (ii) determine their constant value. In this poster we show that credal networks with binary states hold a very general intrinsic monotonicity property in regards to inference. This property can be exploited to drastically reduce the dimensionality of the inference optimisation problem. It was also found that networks with three states or more do not hold the same advantageous property.

References

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