Towards Markov Chain Monte Carlo Methods for Uncertainty Models Specified by Sets of Probability Distributions*

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Numerical treatment of uncertainty models poses some challenges even if the models are specified by a single probability distribution. The main game-changer in the field was the introduction of Monte Carlo algorithms, which provided a general procedure for drawing inferences from a wide range of uncertainty models. But an efficient Monte Carlo algorithm is still absent for imprecise probability models.

Imprecise probability theory models uncertainty as a set of probability distributions, the credal set. Inferences from such models can be drawn by evaluating the lower and upper bounds of the inferences over the credal set. We focus on models which credal sets are specified by sets of parametric probability density functions. A Monte Carlo algorithm for this scenario has already been proposed as an extension of the *importance sampling Monte Carlo* algorithm [1].

We will present some preliminary results on the development of an alternative approach emerging from adapting the *Metropolis-Hastings* algorithm, hence towards utilizing the advances in *Markov Chain Monte Carlo* methods.

The Metropolis-Hastings Markov Chain Monte Carlo scheme, for a precise stochastic uncertainty model, aims to construct a trajectory of a Markov chain, the stationary distribution of which is the targeted probability distribution. Samples from this trajectory can be used to approximate samples from the target distribution and used in the Monte Carlo estimation of quantities of interest. If the uncertainty model is, instead, modelled by a set of probability distributions, application of the Metropolis-Hastings scheme would generally produce a set of trajectories. Each trajectory from this set can be used to construct Monte Carlo estimates of the quantities of interest, hence, altogether, the procedure produces estimates of their lower and upper bounds. We will present such a scheme, which is applicable for a wide range of imprecise probability models and which generates only a finite (yet exponentially large) set of trajectories.

A general step of the Metropolis-Hastings scheme proposes a new sample, which is either accepted or rejected as the new position of the generated trajectory. The new sample is accepted with probability dependent on the target distribution. This is usually implemented by sampling an ancillary uniform random variable and accept the new sample if the ancillary sample is below the acceptance probability. With an imprecise probability model, the distributions in the credal set may either agree whether to accept or reject the new sample, or disagree. The core of our method is that in the case of disagreement, the credal set is divided into a set that accepts the new sample and a set that rejects it, the trajectory of the generated Markov chain branches, and the algorithm proceeds (almost) independently for each of the branches. We will show that this dividing of credal sets can be implemented exactly (the division boundary is a hyper-plane) for sets of distributions from the exponential family and for multi-variate and hierarchical models derived from them.

The main issue at the moment is the large number of generated trajectories, hence the computational complexity of the scheme. Several heuristic approaches will be suggested to mitigate it.

References

[1] Matthias C.M. Troffaes. Imprecise monte carlo simulation and iterative importance sampling for the estimation of lower previsions. *International Journal of Approximate Reasoning*, 101:31–48, 2018. ISSN 0888-613X.

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