# On the Decomposition of Belief Functions Into Simple Support Functions

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Shafer [5] proposed to interpret belief functions as stemming from independent simple support functions (SSF), each representing a partially reliable and elementary testimony. Smets [6] followed in his footsteps and proposed an alternative decomposition of belief functions into independent SSF, arguing that Shafer's was not entirely satisfactory. Smets's proposal is formally elegant and has enjoyed some success; it forms in particular the basis of the cautious rule introduced by Denœux [1]. Nonetheless, it raises its own issues, as discussed recently in [4], where both Shafer's fundamental idea of decomposing belief functions into SSF and Smets's proposal are revisited leading to a new decomposition of belief functions into SSF and a completely different perspective on Smets's proposal. In this poster, the essential aspects of these latter two contributions are presented.

First, it is shown that besides the representation of elementary testimonies having independent reliabilities, the theory of belief functions allows also the representation of elementary testimonies having *dependent* reliabilities. More precisely, whatever the considered set of partially reliable and elementary testimonies (and in particular whatever the dependencies between their reliabilities), there exists a unique belief function representing it, and, more importantly, any belief function can be associated uniquely to a particular set of partially reliable and elementary testimonies inducing it. Moreover, this latter association makes it possible – using both Teugels's representation of the multivariate Bernoulli distribution [7] and Destercke and Dubois's general approach to the conjunctive combination of belief functions [2] – to express uniquely any belief function as a conjunctive combination of some SSF having some dependency structure. These SSF are nothing but the representations of the partially reliable and elementary testimonies in the above-mentioned association, and this dependency structure is nothing but the central moments between the reliabilities of the elementary testimonies. This new decomposition of belief functions into SSF casts a fresh light on belief functions and does not suffer from the criticisms that have been addressed to Shafer and Smets's decompositions.

Secondly, it is shown that instead of interpreting with some difficulty the weight function underlying Smets's proposal as a decomposition of a belief function into SSF, it is possible to give it a different and well-defined semantics in terms of measures of information (specifically, conditional mutual information measures [3]) associated with the reliabilities of the elementary testimonies in our new decomposition.

## References

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