Let's Replace P-Values With Betting Outcomes. *

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Shafer and Vovk [4] locate the meaning of objective probability, precise or imprecise, in its resistance to betting schemes that try to multiply the money they risk. This has surprisingly simple and powerful implications for statistics. Consider, for example, the null hypothesis that a phenomenon Y is described by a probability distribution P.

Test the Null Hypothesis by Betting Against It.

- A 5% significance test is an all-or-nothing bet. You multiply your money by 0 or 20.
- Instead, make a bet that can multiply your money by many different amounts, depending on Y's observed value y. Do this by paying \$1 to get back a nonnegative payoff S satisfying $\mathbf{E}_P(S) = 1$.
- The observed value $S(y)$ is the amount you multiply your money. Call it your *betting score*. A large $S(y)$ discredits P.

Your Bet Implies an Alternative Hypothesis.

- $\mathbf{E}_P(S) = 1$ can be written $\sum S(y)P(y) = 1$.
- So SP is a probability distribution. Call it Q. Then $S(y) = Q(y)/P(y)$. *A betting score is a likelihood ratio!*

Make Meta-Analysis Work. By Gibb's inequality, the bet S maximizes $\mathbf{E}_Q(\ln S)$. Maximizing $\mathbf{E}_Q(\ln S)$ is appropriate because we may multiply the winnings $S(y)$ by investing $\$S(y)$ in a further test of P. As noted by Kelly [2], "it is the logarithm which is additive in repeated bets and to which the law of large numbers applies."

Replace Power with Implied Target. A proposed study using S to test P should be publishable when (1) both P and the implied alternative Q are initially plausible and (2) $\exp(\mathbf{E}_Q(\ln S))$, the betting score targeted by S, is reasonably large.

Proposed study		
initially unknown outcome	phenomenon	Y
probability distribution for Y	null hypothesis	P
nonnegative function of Y with expected value 1 under P	bet	S
SP	implied alternative	Q
$\exp(\mathbf{E}_Q(\ln S))$	implied target	S^*
Results		
actual value of Y	outcome	y
factor by which money risked has been multiplied	betting score	$S(y)$

Extend the Theory to Composite Null Hypotheses. See Grünwald [1].

References

[1] Peter Grünwald. Safe testing, 2019. In preparation.

[2] John L. Kelly Jr. A new interpretation of information rate. *Bell System Technical Journal*, 35(4):917–926, 1956.

[3] Glenn Shafer. The language of betting as a strategy for statistical and scientific communication. 2019. Working Paper 54 at www.probabilityandfinance.com. Also at arXiv:1903.06991 [math.ST].

[4] Glenn Shafer and Vladimir Vovk. *Game-Theoretic Foundations for Probability and Finance*. Wiley, 2019.

* This proposal emerges from 25 years of work with Vladimir Vovk, summarized in [4]. See [3] for details and a treatment of statistical estimation.