A New Class of Multivariate Prior Distributions with an Application to Reliability Engineering *

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The key of the success of Bayesian statistics lies in its ability to incorporate prior knowledge about the quantity of interest as a distribution function. This prior distribution, together with experimental data, leads in general to a better estimation of the quantity under study. A thorough review of the Bayesian approach can be found in **?**]. As it is nicely described in the literature, any elicitation process leading to prior information is to some extent arbitrary. This is exactly one of the main problems addressed by robust Bayesian analysis, also called Bayesian sensitivity analysis, quantifying and interpreting the uncertainty induced by the partial knowledge of the prior information.

Focusing on the prior uncertainty, it is a common practice in the literature to replace the specific prior distribution by a class of priors Γ. In this context and considering multiparameter distributions, we introduce a new class of priors based on notions from different fields in Statistics. Let $\mathbf{X}$ and $\mathbf{Y}$ be two random vectors with PDFs f and g, respectively, such that

$$f(\mathbf{x})g(\mathbf{y}) \leq f(\mathbf{x}\wedge\mathbf{y})g(\mathbf{x}\vee\mathbf{y}), \text{ for every } \mathbf{x} \text{ and } \mathbf{y} \text{ in } \mathbb{R}^n.$$

Then $\mathbf{X}$ is said to be smaller than $\mathbf{Y}$ in the likelihood stochastic order, denoted by $\mathbf{X} \leq_{lr} \mathbf{Y}$, where $\mathbf{x}\vee\mathbf{y}$ and $\mathbf{x}\wedge\mathbf{y}$ mean the componentwise maximum and the componentwise minimum, respectively. A function $l : \mathbb{R}^n \mapsto \mathbb{R}^+$, $(n \in \mathbb{N}, n \geq 2)$ is said to be multivariate totally positive of order 2 (MTP_2), ($TP2$ when $n = 2$) if it satisfies

$$l(\mathbf{x})l(\mathbf{y}) \leq l(\mathbf{x}\wedge\mathbf{y})l(\mathbf{x}\vee\mathbf{y}),\ \forall \mathbf{x},\mathbf{y} \in \mathbb{R}^n.$$

Additionally, a n-dimensional random vector $\mathbf{X}$ with PDF f is said to be MTP_2 if its density f is MTP_2. Given a prior π with density function $\pi(\theta)$, $\theta \in \Theta \subseteq \mathbb{R}^n$, let ω be a weight function, i.e, a non-negative function $\omega : \mathbb{R}^n \mapsto \mathbb{R}^+$ such that the expectation $E^{\pi}[\omega(\theta)]$ is strictly positive and finite. A weighted random vector π_{ω} is a random vector with density function given by

$$\pi_{\omega}(\theta) = \frac{\omega(\theta)}{E^{\pi}[\omega(\theta)]}\pi(\theta), \quad \forall\theta \in \Theta \subseteq \mathbb{R}^n.$$

Let π be a specific MTP_2 prior belief. We will define the weighted band $\Gamma_{w_1,w_2,\pi}$ associated with π based on w_1 and w_2, a decreasing weight function and an increasing weight function, respectively, (weighted band, for short), as

$$\Gamma_{w_1,w_2,\pi} = \left\{\pi' : \pi_{\mathbf{w_1}} \leq_{\mathbf{lr}} \pi' \leq_{\mathbf{lr}} \pi_{\mathbf{w_2}}\right\}.$$

This new class is a multivariate generalization of a class of priors recently introduced in **?**]. We will study its main properties and the relationship with other classical classes of prior beliefs as the contamination class and the distribution band. We also consider the Hellinger metric and the Kullback-Leibler divergence to measure the uncertainty induced by such a class, as well as its effect on the posterior distribution. Finally, we will show a real example about train door reliability.

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