Exposing Some Points of Interest About Non-Exposed Points of Desirability

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Sets of desirable gambles [6, 4, 3] are a very general and elegant framework to model uncertainty. A set of desirable gambles D is a set of gambles—which are real-valued maps on the finite possibility space Ω—that the subject strictly prefers to the status quo indicated by 0. The set of all gambles is denoted by $\mathscr{L}$. We say that D is coherent if it is a convex cone that does not contain 0 and contains the positive gambles $\mathscr{L}_{>0} := \{f \in \mathscr{L} \colon f > 0\}$, where we define $f > 0 \Leftrightarrow (\forall \omega \in \Omega) f(\omega) \geq 0$ and $f \neq 0$. Coherent sets of desirable gambles are more general than convex sets of probabilities, even when these convex sets are not required to be closed: indeed, they do not have an Archimedean condition and are therefore not representable by real-valued standard probabilities.

Recently, Cozman [1] has given an axiomatisation for sets of desirable gambles that make them uniquely representable by a convex, but not necessarily closed, set of probabilities. He shows that any *evenly convex* coherent set of desirable gambles—that is, a coherent set of desirable gambles that is an arbitrary intersection of affine open semi-spaces—is uniquely represented by a convex set of probabilities, and gives an elegant equivalent requirement in terms of gambles.

More than 20 years earlier, in 1995, Seidenfeld et al. [5] gave an axiomatisation of binary preferences that leads to a unique representation of convex sets of probabilities. Since binary preferences are closely related to sets of desirable gambles, Seidenfeld et al. [5]'s requirement must be similar to that of even convexity. There is however a difference: Seidenfeld et al. [5]'s options between which the subject must state his preferences, are horse lotteries, instead of gambles, but Cozman [1] has shown that their ideas can be straightforwardly used for gambles as well. Roughly speaking, and after translating to sets of desirable gambles, what Seidenfeld et al. [5] show, is, amongst other things, that any coherent set of desirable gambles that (i) satisfies an Archimedean axiom, which we will refer to as 'SSK-Archimedeanity', in the same vein as Cozman [1], and (ii) is the result of a particular extension, which we will refer to as 'SSK-extension', is uniquely represented by a convex set of probabilities.

Interestingly, in his paper, Cozman [1] shows that SSK-Archimedeanity is not sufficient for even convexity. He does so by providing an explicit example of a coherent and SSK-Archimedean set of desirable gambles that is not evenly convex. In this poster, we will expand on this connection between SSK-Archimedeanity, SSK-extension, and even convexity. We will show the extent of SSK-Archimedeanity more precisely, and argue that there are no other types of coherent sets of desirable gambles that are SSK-Archimedean but not evenly convex than the type of Cozman [1, Example 17]. Finally, we will argue that the combination of SSK-Archimedeanity and SSK-extension is equivalent to even convexity.

References

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