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DecideIT 3.0: Software for Second-Order Based Decision Evaluations

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Abstract

In this paper, we discuss representation and evaluation in the DecideIT 3.0 decision tool which is based on a belief mass interpretation of the background information. The decision components are imprecise in terms of intervals and qualitative estimates and we emphasise how multiplicative and additive aggregations influence the resulting belief distribution over the expected values.

Keywords: Belief distribution, decision analysis, bounded Dirichlet, skew-normal distribution

1. Introduction

There have been many suggestions for how to deal with the strong requirements of most decision models to provide precise information, such as the theory of capacities, probabilistic logic, sets of probability measures, interval probabilities, evidence and possibility theories, fuzzy measures, preference rankings, extended elicitation methods and higher-order probability theory (see for example [2], [14], [5], [6], [12], [20], [3], [16], [19], [13] to name just a few in the extensive literature in the fields). Often, these theories require significant mathematical knowledge on the part of the decision-maker, and sometimes include relatively harsh methods for discriminating between decision alternatives. Furthermore, the computational complexity can be high in various respects, as argued in some of our earlier work (for an extensive background, see e.g. [7], [4], [10]. The purpose of this paper is to discuss the ideas behind the interactive software tool DecideIT 3.0 and the underlying framework for evaluations under risk subject to incomplete input data and to briefly showcase it. The software is able to evaluate decision situations specified by imprecise utilities and probabilities and qualitative estimates between these components. It is a substantial improvement over earlier versions of the program, see, e.g., [9], and it has recently been used in large scale projects for energy policy modelling in Jordan and Morocco, cf. [11], [8], [17]. A key idea is the use of higher-order distributions of belief, which allows for better discriminating between the alternatives in the decision tree.

2. Representation

The components of a decision tree T are a root node (also called a decision node), a set of probability nodes (representing uncertainty) and consequence nodes (the final outcomes). The probability and consequence nodes are normally assigned unique probability and value distributions, but this can of course be generalised to cases where there is imprecise or incomplete information with respect to probabilities and consequence or alternative values. User statements may be range constraints or comparative statements, which are translated into systems of inequalities in a constraint set. Probability and utility statements are collected in a node constraint set. User statements have the forms of range constraints, e.g., a probability or value y_i lies between a_1 and a_2, and comparisons: y_i is larger than y_j by a difference from d_1 to d_2. When specifying an interval, the actual beliefs in the values are presumably not uniformly distributed. One way to formalise this is to introduce belief distributions that indicate the strengths with which we believe in these different values. We use different distributions for probabilities and values because of the normalisation constraints for probabilities; natural candidates are the Dirichlet distribution for probabilities and two- or three-point distributions for values. The properties of the Dirichlet distribution as a parameterised family of continuous multivariate probability distributions makes it suitable for this purpose. We use a slightly different form, namely the bounded Dirichlet distribution over a (normally user-specified) range instead of the interval $[0,1]$ and parameterised so that the distribution of belief is uniform over the simplex. Bounded beta distributions are then derived from this, yielding four-parameter beta distributions. Thus, we define a probability belief distribution through a bounded Dirichlet distribution $f(a_i, c_i, b_i)$ where c_i is the estimated most likely probability and where a_i and b_i are the boundaries for the support of the distribution ($a_i < c_i < b_i$) (cf. [18]). For the utilities/values (i.e. without normalisation constraints), the generalisation to trapezoids is straightforward. The baseline distribution is a two-point distribution (uniform or trapezoidal) which can be extended to a three-point distribution (triangular). When a decision-maker has no reason to make any other specific assumptions, for instance when there is large uncertainty in the underlying belief distributions involved, a two-point distribution modelling the upper and lower bounds (the uniform or trapezoid distributions) seems to be reasonable. Rather, when modal outcomes can be estimated to some extent, the beliefs are better represented by three-point distributions. Popular distributions like Beta and Erlang generally give rise to results similar to triangular distributions. For instance, [15] considers some frequently employed distributions. Here, we assume that we only have limited sample data, essentially the minima, maxima, and modal values. The mean value of a number of three-point value belief distributions $f(a_i, c_i, b_i)$ is $\mu(\lambda) = (a_i + b_i + \lambda c_i) / (\lambda + 2)$, where Beta usually employs $\lambda = 4$ and Erlang employs $\lambda = 3$, with $\lambda = 1$ for triangular distributions and $\lambda = 0$ for a two-point uniform or trapezoid distribution as special cases. For practical real-life purposes, there is usually no reason to use any three-point distribution other than a triangular distribution since the risk of underestimation is kept lower.

3. Evaluation

The evaluation model is based on the resulting belief distribution over the generalised expected utility, i.e., given a decision tree T and an alternative A_i the expression

$$E(A_i) = \sum_{i_1=1}^{n_{i_0}} p_{ii_1} \sum_{i_2=1}^{n_{i_1}} p_{ii_1 i_2} \cdots \sum_{i_{m-1}=1}^{n_{i_{m-2}}} p_{ii_1 i_2 \ldots i_{m-2} i_{m-1}} \sum_{i_m=1}^{n_{i_{m-1}}} p_{ii_1 i_2 \ldots i_{m-2} i_{m-1} i_m} u_{ii_1 i_2 \ldots i_{m-2} i_{m-1} i_m}$$

(where m is the depth of T corresponding to A_i, n_{i_k} is the number of possible outcomes following the event with probability p_{i_k}, $p_{\ldots i_j \ldots}$, $j \in [1, \ldots, m]$ denote probability variables, and $u_{\ldots i_j \ldots}$ denote utility variables) is the *expected utility* of alternative A_i in T.

Note that even when we assume that the expectations are estimated a large number of times (due to repeated decision making, such as the assumption of going concern in business administration) and can consequently be approximated by a normal distribution, there are three observations in particular that should be considered:

- The resulting distributions will be approximately normal only when the original distributions are symmetric, which of course is not usually the case for beta and triangular distributions. The result will instead be skew-normal.
- Even if the original distributions are symmetric, the non-linear multiplication operator breaks the symmetry. The result will again be approximately skew-normal.
- To obtain a resulting approximate normal distribution, both the original distributions and their aggregations must allow for long tails. In general, this is not the case here; the estimates have lower and upper boundaries due to the fact that we use bounded Dirichlet distributions and uniform and triangular distributions, yielding approximately truncated normal distributions.

We therefore employ skew-normal distributions to generalise the normal distribution of belief by allowing for non-zero skewness, i.e. asymmetry. This is accomplished in the traditional way by introducing a shape parameter α, where $\alpha = 0$ represents the standard normal distribution, and $\alpha = 1$ yields the distribution of the maximum of two independent standard normal variates. We can then conveniently represent truncated (skew-)normal distributions as probability distributions of (skew-)normally distributed random variables that are bounded. The skewness of the distribution increases with the absolute value of α, and when $|\alpha| \to \infty$, we get folded normal or half-normal distributions. Distributions are right-skewed when $\alpha > 0$ and left-skewed when $\alpha < 0$. When the sign of α is changed, the density is reflected about $x = 0$. The skew-normal density function with location ξ, scale ω, and shape parameter α is

$$f(x) = \frac{2}{\omega} \varphi\left(\frac{x-\xi}{\omega}\right) \Phi\left(\alpha\left(\frac{x-\xi}{\omega}\right)\right)$$

The use of a skew-normal belief distribution together with the three observations above and the principle of going concern is called the B-normal (business normal) method. By this assumption, there is no need to select case-specific distributions in modelling decision situations. To employ the B-normal method, the skewed distribution must be aligned to give the same variance and expected value as its unskewed counterpart and must display the correct shape (skew). Assume that the desired expected value is $E(X)$, the desired variance is σ^2, and the desired skew is s. The alignment (matching) of the B-normal distribution is then done in three steps:

- Obtain the shape parameter α that describes the desired skew s of a skew-normal distribution. The user specifies the shape indirectly by indicating the skew of the individual distributions, since entering a most likely (modal) number in addition to minimum and maximum numbers indirectly indicates the skew (asymmetry) for each variable. This skew is subsequently propagated along the decision tree;
- Once the shape parameter α is determined, this changes the variance of the B-normal distribution com-

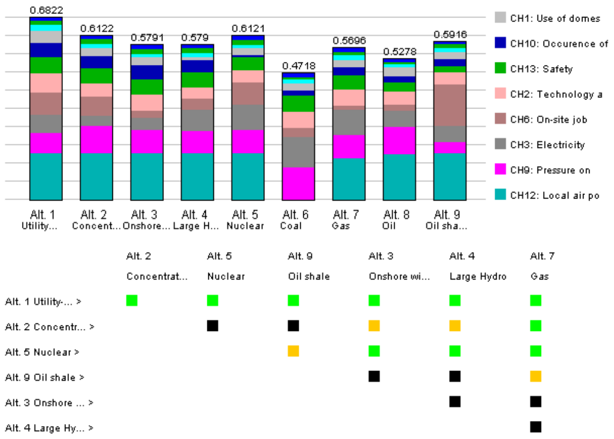

Figure 1: Pairwise comparison of alternatives

pared to a normal distribution. Adjust the scale parameter ω until the variance of the B-normal distribution is σ^2 and thus coincides with the corresponding normal distribution;

- When the shape and variance have been determined, this in turn changes the expected value of the distribution. To obtain the desired expected value $E(X)$, use the standard formula for the mean of a skew-normal distribution and solve for the location parameter ξ.

This procedure will yield the parameters α, ω^2, and ξ, and once these have been obtained the B-normal distribution is parametrically determined. From this distribution, the belief (or confidence) in the different expected values can be determined in the same way as with standard normally distributed information. Together with the principle of going concern, in which decision-making is carried out on a repeated basis, this is the core of the method that underlies the evaluations. The distribution of belief in expected values resulting from the decision-maker's input is conveyed to the user via the GUI (graphical user interface).

4. The GUI

Among the main additions for DecideIT 3.0 are the functionality to enter ordinal and cardinal rankings interpreted using surrogate weights [8] and the evaluation of expected values by the distribution of belief in addition to the earlier distribution-free evaluations [9]. The assessments can be entered in the form of fixed numbers, intervals with or without modal points, and ordinal or cardinal rankings. If desired, the decision situation can be modelled using several criteria, with each criterion potentially containing a decision/event tree. Figure 1 is from the analyses in the project [17] and shows a comparison of all alternatives with a pairwise ranking based on the belief in superiority (higher expected value) for each of the two alternatives in each pair. A green square represents that at least 90% of the belief mass resides with the horizontal alternative, and a yellow one between 75 – 90%. This way, a ranking of all alternatives is obtained. The software is available for free for academic and other non-commercial purposes [1].

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