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On Intercausal Interactions in Probabilistic Relational Models

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Abstract

Probabilistic relational models (PRMs) extend Bayesian networks beyond propositional expressiveness by allowing the representation of multiple interacting classes. For a specific instance of sets of concrete objects per class, a ground Bayesian network is composed by replicating parts of the PRM. The interactions between the objects that are thereby induced, are not always obvious from the PRM. We demonstrate in this paper that the replicative structure of the ground network in fact constrains the space of possible probability distributions and thereby the possible patterns of intercausal interaction.

Keywords: PRM instances, qualitative constraints on probability distributions, intercausal interaction.

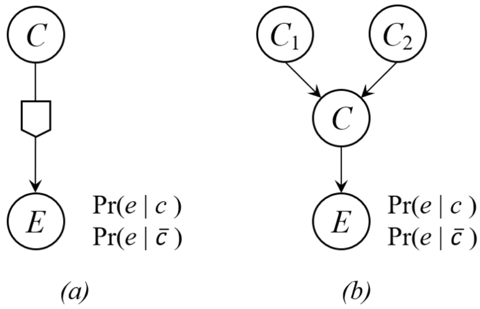


Figure 1: *(a)* An example PRM and *(b)* its ground network.

1. Introduction

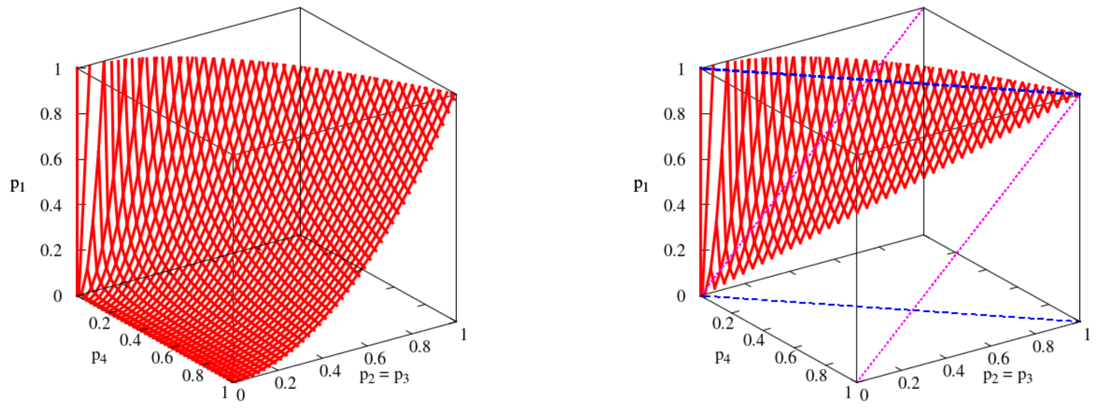
Real-world problem domains often have a relational structure involving multiple interacting object classes, which cannot be appropriately modelled by a Bayesian network. By allowing the representation of relational information, *probabilistic relational models*, or *PRMs*, extend on the propositional expressiveness of Bayesian networks [2, 3, 4]. More specifically, a PRM describes the object classes from a relational schema by a graphical dependency structure and supplements this structure with probabilistic information. Inference with an *instance* composed of sets of concrete objects per class is performed in a *ground Bayesian network* which is derived from the PRM by replicating parts for the concrete objects involved. The PRM thus is essentially a *meta-model* covering all possible instances.

Upon constructing a ground Bayesian network for a given instance, interactions may be induced between the random variables modelling the PRM's feature variables, that are not always obvious from the meta-level model. In this paper, we study the possible interactions resulting for an instance from relational dependencies between the features of different classes. We focus more specifically on *many-to-one* dependencies: for a given instance, the value of a feature variable E of a concrete object for some class X may then depend on the values of the variable C in *multiple* objects for a class X'. In the PRM, the precise dependency involved is described by an *aggregation function* which serves to summarise a multiset of values of C into a single such value [3]. Example aggregation functions are the logical OR, the MODE and the STOCHASTIC-MODE [5]. In this paper, we will demonstrate that, in addition to the aggregation function used, the replicative structure of the ground network constrains the intercausal interactions induced among multiple objects. Our analysis informs the choice of aggregation function to attain the desired type of interaction in a PRM instance.

2. Running Example

As running example, we consider the small PRM shown in Figure 1*(a)* and assume all variables to be binary-valued. The feature variables C and E belong to different classes (left implicit). The relational dependency between C and E thus extends over the class boundaries, and we assume it to be of type many-to-one, indicated by an arc associated with an aggregation function. We now consider an instance with two objects with the variable C, whose replicates are denoted C_1 and C_2 as shown in Figure 1*(b)*. For the node C, encoding the aggregation of the two C_i values, in essence four conditional probabilities are specified:

$$p_1 = \Pr(c \mid c_1, c_2) \qquad p_3 = \Pr(c \mid \bar{c}_1, c_2)$$
$$p_2 = \Pr(c \mid c_1, \bar{c}_2) \qquad p_4 = \Pr(c \mid \bar{c}_1, \bar{c}_2)$$



(a) *(b)*

Figure 2: *(a)* Combinations of probability values p_1, $p_2 = p_3$, p_4 resulting in $X^0(\{C_1,C_2\},c)$ (above the surface the synergy is positive, and it is negative below the surface), *(b)* the $X^0(\{C_1,C_2\},c)$-surface restricted to combinations of probability values $p_1 \geq p_2 = p_3$ and $p_2 = p_3 \geq p_4$ obeying $S^+(C_i,C)$.

Since the ground Bayesian network of Figure 1*(b)* has a replicative structure, we necessarily have that $p_2 = p_3$ to capture an arbitrary aggregation order.

3. Intercausal Interactions

We address the patterns of interaction between the two replicates C_1 and C_2 in our running example, that is, we investigate how an observation for C_1 influences the probability distribution over C_2, and vice versa, when a value for E is known; without loss of generality, we assume in our analysis that a value is known for the aggregation node C instead. The sign of such an intercausal influence is described by the concept of *product synergy* [8]. More specifically, a product synergy $X^\delta(\{C_1,C_2\},c)$ captures the sign $\delta \in \{+,-,0\}$ of the intercausal interaction between C_1 and C_2 that is induced by observation of $C = c$, where δ is the sign of the difference $p_1 \cdot p_4 - p_2 \cdot p_3$. If $\delta = 0$ then there is no interaction between the causes upon observing $C = c$; if $\delta = -$, then the presence of one cause is said to *explain-away* the other.

We exploit the above and similar qualitative properties of probability to show that not all patterns of intercausal interaction can be readily attained in a ground Bayesian network. Properties of the PRM in fact dictate to a large extent the set of possible probability values p_i, $i = 1,2,3,4$, for the aggregation node C, and thereby the patterns of interaction that can be induced. Given the constraint $p_2 = p_3$, Figure 2*(a)* shows the surface of all p_i combinations that result in a zero product synergy, and hence in a lack of interaction between C_1 and C_2 given $C = c$. Probability combinations above this surface result in $X^+(\{C_1,C_2\},c)$ and combinations below it in explaining-away. To aid in the interpretation of Figure 2*(a)*, Figure 3 shows two slices of the surface: the left one is the horizontal slice for $p_1 = 0.8$, and the right one the vertical slice at $p_2 = p_3 = 0.5$.

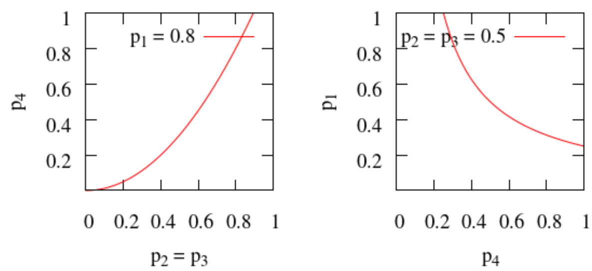


Figure 3: Slices of the $X^0(\{C_1,C_2\},c)$-surface for fixed values $p_1 = 0.8$ and $p_2 = p_3 = 0.5$, respectively.

Another qualitative property of probability distributions is described by the concept of *qualitative influence* [7], denoted by $S^\delta(C_i,C)$, which captures the sign of the direct influence between two variables C_i and C. More specifically, the influence is positive iff $\Pr(c \mid c_i, C_j) \geq \Pr(c \mid \bar{c}_i, C_j)$ for all values of C_j, $i,j = 1,2$, $i \neq j$; assuming $c > \bar{c}$ and $c_i > \bar{c}_i$, such an influence captures the idea that higher values of C_i make higher values of C more likely, regardless of other influences on C. In the ground Bayesian network, the direct influences are constrained by $S^+(C_i,C)$, $i = 1,2$ [6], that is, the set of possible probability values p_i, $i = 1,2,3,4$, is restricted by $p_1 \geq \{p_2,p_3\} \geq p_4$. The plane representing the constraint $p_1 = p_2 = p_3$ is shown by dotted lines in Figure 2*(b)*; dashed lines show the plane representing the constraint $p_2 = p_3 = p_4$. The figure shows that these constraints significantly reduce the possible p_i combinations that induce a zero product synergy, and leave very little room for capturing positive intercausal interactions in a PRM at hand. We further note that any deterministic aggregation function, encoded by a degenerate distribution for C with $p_1 = 1$ and $p_4 = 0$, will necessarily result in negative intercausal interactions given $C = c$. If positive intercausal interactions or a lack of interaction are desired therefore, the use of a stochastic aggregation function is called for.

The product synergies discussed capture the intercausal interaction upon observation of $C = c$. The intercausal interactions induced by observing $C = \bar{c}$ are found basically by mirroring the surface across all three axes in the figures.

4. Conclusion and Future Research

Our analyses show that to induce the type of intercausal interaction desired for an application domain requires careful tuning of the probabilities for the observed value of aggregation node C. As the probabilities for this node in essence encode an aggregation function, studying the possible probability values for C thus informs the choice of an appropriate aggregation function for the PRM.

In view of possible instantiations of the PRM we are typically interested in intercausal effects due to the observation of the effect node E rather than aggregation node C. An observation for node E is essentially indirect evidence for C in the ground Bayesian network, and capturing the induced intercausal effects then involves the product synergies for both values of C [1]. For instantiations with more than two objects, we can use a generalised definition of product synergy to similarly study all pair-wise interactions [1]. The concepts of product synergy and qualitative influence are not restricted to binary variables, but are in fact defined for discrete variables that allow a total order on their values. By considering all possible pairs of values $c_i > c_j$ of the variables involved, our results generalise straightforwardly beyond the binary case. A more formal underpinning of further generalisations is topic of future research.

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