Modeling higher order evidence using imprecise probabilities

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A puzzle from mainstream epistemology

Higher order evidence (HOE) is evidence which does not concern the primary subject matter of your beliefs but rather concerns your ability to process certain first-order evidence rationally or reliably. You may for instance learn that your reasoning is compromised in some way, or that others in similar circumstances have not been capable of making reliable inferences. Is there a rational response to such HOE, and if so, what is it? How should you revise your beliefs on receiving HOE?

Cases of HOE

Cases of HOE typically involve discovering that you are less reliable than you thought. But the evidence of your (un)reliability can take different forms. It may be quite specific to a particular proposition, or more of a general reason to doubt your own capacities.

Doctor

Anton is an anesthesiologist, trying to determine which dosage of pain medication is best for his patient: A or B. To figure this out, Anton assesses some fairly complex medical evidence. When evaluated correctly, this kind of evidence determines which dose is right for the patient. After thinking hard about the evidence, Anton becomes highly confident that dose B is right. In fact, Anton has reasoned correctly; his evidence strongly supports that B is the correct dose. Then Sam, the chef at the hospital's cafeteria, rushes in. "Don't administer that drug just yet", he says guiltily. "You're not in a position to properly assess that medical evidence. I slipped some reason-distorting mushrooms into your frittata as a prank. These mushrooms make you much less reliable at determining which dose the evidence supports: in the circumstances you presently face – evaluating this type of medical evidence, under the influence of my mushrooms – doctors like you only tend to prescribe the right dose 60% of the time!". In fact, Sam is mistaken: the mushrooms he used were just regular dried porcini, and Anton's reasoning is not impaired in the least. But neither he nor Anton knows (nor has reason to suspect) this.

(Sliwa and Horowitz (2015) 'Respecting all the evidence', *Phil. Stud.*)

Hypoxia (version 1)

Aisha is flying her airplane on a bright Monday morning, wondering whether she has enough gasoline to fly to Hawaii. Upon looking at the dials, gauges and maps, she obtains some first order evidence E, which she knows strongly supports (say to degree 0.99) either that she has enough gas (G) or that she does not have enough gas (~G). Aisha does some complex calculations and concludes G, which is, in fact, what E supports. But she then gains some higher order evidence: she realizes that she is flying at an altitude that puts her at great risk for hypoxia, a condition that impairs one's reasoning capacities. Aisha knows that pilots who do the kind of reasoning that she just did, and who are flying at her current altitude, only reach the correct conclusion 50% of the time.

(Schoenfield (2018) 'An accuracy based approach to higher order evidence', *PPR*)

Hypoxia (version 2)

I have just achieved a difficult first ascent in the Himalayas. As the weather turns, I have to abseil down a long pitch. I have gone through a sequence of reasoning several times to check that I have constructed my anchor correctly, that I haven't under-estimated the length of the pitch, and that I have threaded the rope correctly through my belay device and carabiner. I then acquire evidence that I am in serious danger of being affected by a mild case of hypoxia caused by high altitude. Such hypoxia impairs one's reasoning while making it seem perfectly fine. I know that mountaineers have made stupid but fatal mistakes in the past as a result of being in such a condition.

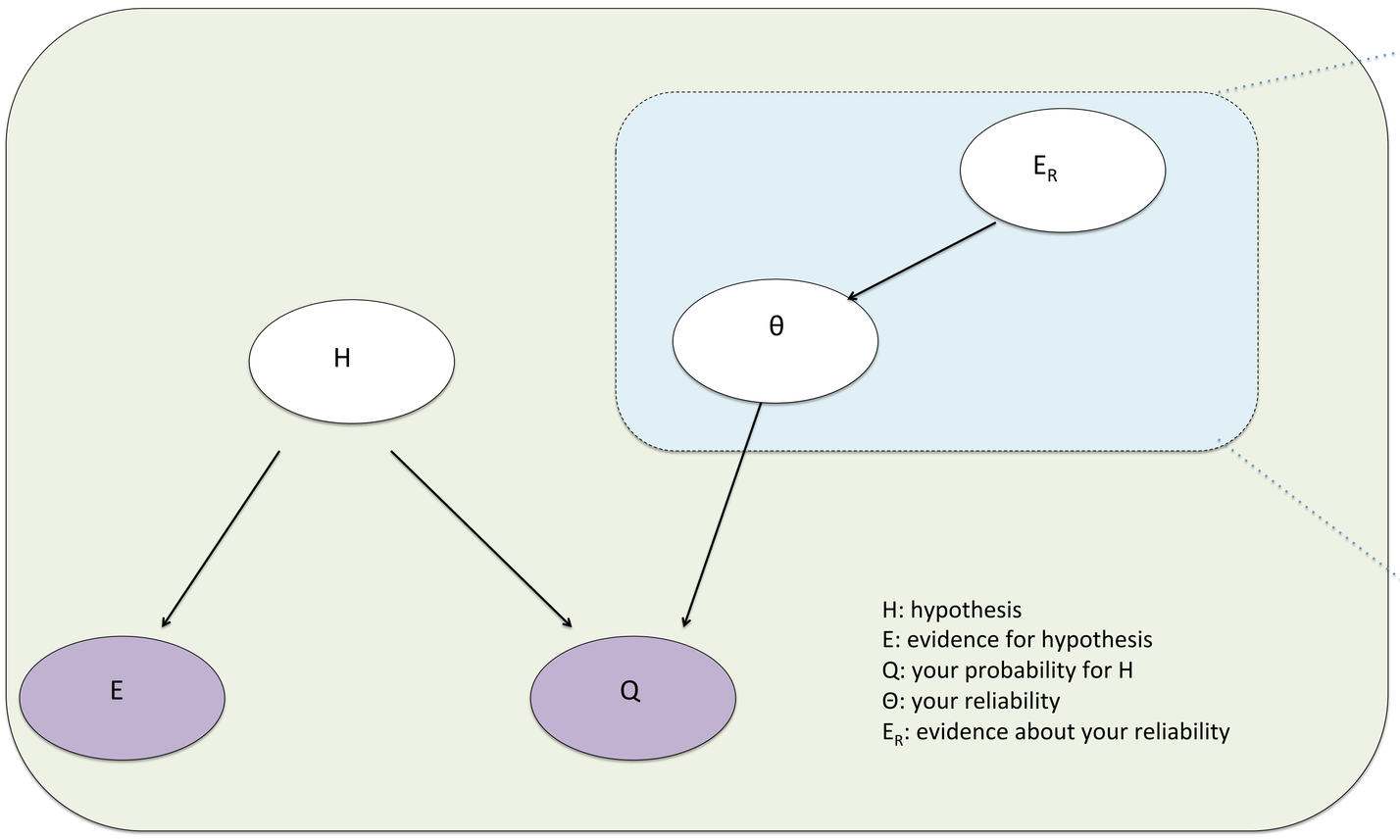
(Lasonen-Aarnio (2014) 'Higher order evidence and the limits of defeat', *PPR*)

Proposal

A framework using imprecise probabilities is suitable for modeling cases of HOE because:

- HOE can induce doubt about your own opinion, not just about first-order matters. This doubt can be represented by the imprecision of your credal state.
- HOE should induce reluctance to make an immediate decision based on your belief. Greater imprecision often leads to choosing the option of not deciding, where that option is available.

MODEL 1: Updating on information from analogous individuals



Prior-data conflict

When there is uncertainty about how relevant the new data are, a strong disagreement between prior and new data may give rise to increased imprecision in θ. See Walley 1991, pp. 217-226, for the following simple model:

$h(\theta) \propto \theta^{st-1}(1-\theta)^{s(1-t)-1}$ Beta prior

$bin(\theta) \propto \theta^{m}(1-\theta)^{n-m}$ Bernouilli likelihood

$R(s_0, t_0) \longrightarrow R(s_n, t_n)$

Initial region

$\underline{s_0} < s_0 < \overline{s_0}$

$\underline{t_0} < t_0 < \overline{t_0}$

Final region

$\underline{s_n} < s_n < \overline{s_n}$

$\underline{t_n} < t_n < \overline{t_n}$

Degree of imprecision in s_0 expresses uncertainty about how seriously to take analogy

Degree of imprecision in t_n is made up of two components: first arising from lack of information, and second from prior-data conflict.

MODEL 2: Discounting

When the HOE casts doubt on the reliability of the agent in general rather than for any specific proposition(s), one might use a `discounting' operation. Discounting involves making a convex combination of the original belief (with weight 1-α) and the fully vacuous belief state [0,1] (with weight α). This represents the situation where there is some chance that the agent is completely mistaken. (cf. Shafer 1976, Mercier et al. 2008, Moral 2018).

Further questions

- Imprecise probability (IP) representations of disagreement cases of HOE have already been proposed (eg. Elkin and Wheeler 2018). Is there a unified way to model disagreement cases of HOE and non-disagreement cases?
- In many cases, receiving HOE should prompt an inclination to gather information. But there are difficulties for IP in relation to free evidence (as reviewed in Bradley and Steele (2016)). Is this a disadvantage of an IP representation of HOE?

References

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Acknowledgements

Thanks to members of the Groningen epistemology reading group and to Teddy Seidenfeld for discussions. This work was funded in part by a Veni grant from the Dutch Science Foundation and by EU support for the Rosalind Franklin Fellowship.