A *possibilistic* interpretation of ensemble predictions: Experiments on the imperfect Lorenz 96 model

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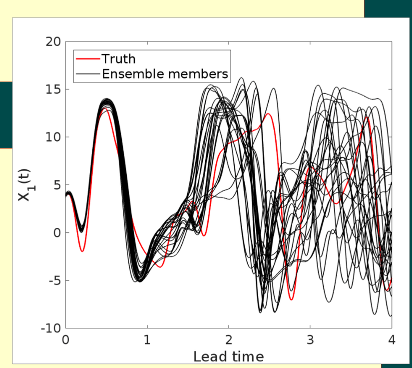
1. Motivations

- **E**nsemble weather predictions assume that the model error is dominated by initial condition (IC) error, hence a Monte-Carlo like sampling of ICs that are then run forward through the model. This assumption is shown not to be true in practice. A PDF estimated from ensemble members (EMs) shows more about the behaviour of the model than about the real system.
- **T**he extremely high dimensionality of the weather phase space makes it in practice mpossible to sample randomly ICs: methods selecting the fastest growing perturbations are used instead, to assess 'all' possible scenari.
- **E**xtreme events (EE) generally result from nonlinear interactions at small scale, which makes them hardly obvious in a probabilistic interpretation of ensemble forecasts.
- **T**he probabilistic interpretation of ensemble predictions consequently generally does not work well without statistical post-processing [4].
- **I**t consists most often in fitting local PDFs modelling the uncertainty on each member, and summing all of them to get a global PDF, supposed to estimate the location of the true system in the phase space (Bayesian model averaging, Best member dressing); or in assuming a parametric form for the global PDF and deriving its parameters from linear combinations of the ensemble's characteristics (mean and variance), e.g. Non-homogeneous Gaussian regression.
- **P**ost-processing improves the ensemble skills for common events and extends the skillfull prediction horizon. However it is shown to deteriorate significantly performances for EE.

2. Problem & Approach

- **A** probabilistic approach of mono-model ensemble prediction systems (EPS) fails to predict events that are not associated with a substantial density of EMs, which is often the case with EEs.
- **W**e need something traducing the possibility of having the system in other areas that the one actually identified by EMs, and yet acknowledging higher probabilities resulting from local agglomeration of EMs: possibility theory, with the combination of dual possibility/necessity functions seems appropriate.
- **C**ontrary to the current probabilistic interpretation (under model error, and biased sampling), a possibilistic development makes more physical sense and offers theoretical guarantees.
- **W**e use it for bounding the probability of a given weather event (here EE) .

4. Test bed & Results



- We reproduce the experiment on an imperfect Lorenz 1996 model developed in [3]. The L96 system was developed as a surrogate model for the atmospheric dynamics. The system dynamics is governed by the following coupled equations:

$$\frac{dX_j}{dt} = X_{j-1}(X_{j+1} - X_{j-2}) - X_j + F - \frac{hc}{b}\sum_{k=1}^{K} Y_{j,k}$$

$$\frac{dY_{j,k}}{dt} = cbY_{j,k+1}(Y_{j,k-1} - Y_{j,k+2}) - cY_{j,k} + \frac{hc}{b}X_j$$

$$0.262 - 1.262X_j + 0.004608X_j^2 + 0.007496X_j^3 - 0.0003226X_j^4$$

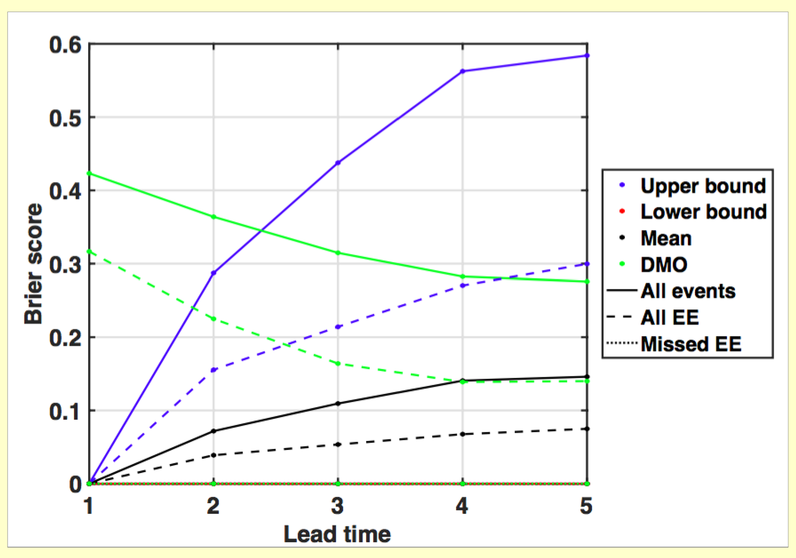
J = 8, K = 32
h = 1, b = 10
C = 10, F = 20

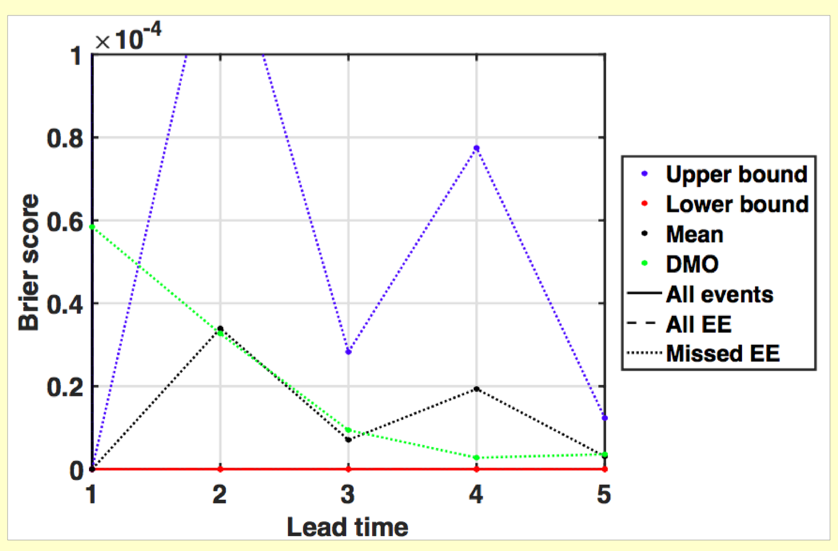
Imperfect model

- $X_1$1 is the variable of interest for prediction. X_j are randomly and independently drawn from $N(X_j, 0.1^2)$
- We use a dataset of 2000 ensemble predictions associated to exact observations for the training of the parameter(s) of the membership function associated to $M^+(m)$: here, a unique (for exchangeable EMs) symmetrical triangular function.
- The objective function consists in minimising the Brier score, here computed from the average of our probability interval:

$$\text{Brier}=\frac{1}{N}\sum_{i=1}^{N}\left(p_i - I(V_i < V_q)\right)^2$$

- We compare our results to those given by the direct model output (DMO) probabilistic prediction:

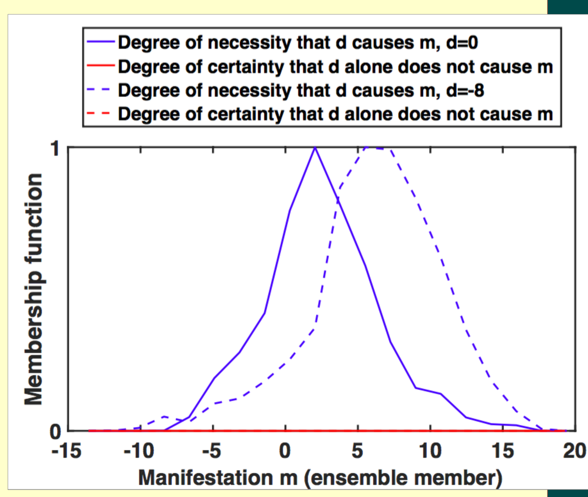




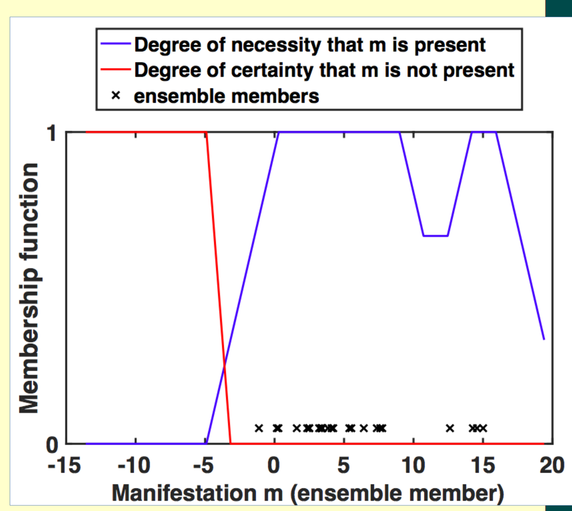
- Results for the quantile q=0.1. The Brier score is lower (mean and lower bound of our probability interval) than the DMO's. The approach is especially more interesting for the prediction of EE at small-medium lead times.
- **Future works** include to combine it to other rules (based on 'data analysis', to identify precursors to system behaviours) in order to get sharper bounds.

3. Methodology

- We use the possibilistic FMECA (fault mode effect analysis) presented in [1]. The EMs $X^m(t)$, $m=1\ldots M$ are manifestations of a disorder $X(t)$, that is the true system state at time t.

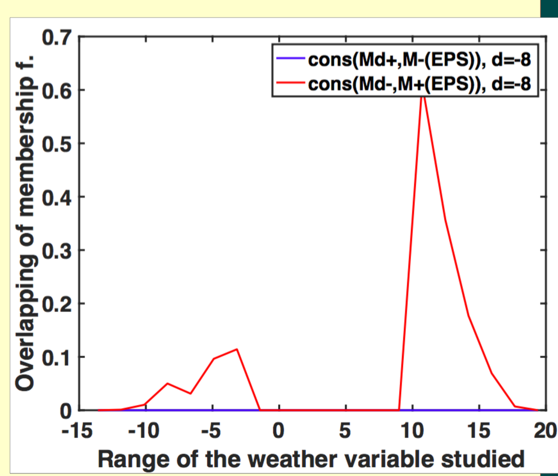


- Each manifestation m is characterized via the twofold fuzzy set $(M^+(m), M^-(m)^C)$, whose respective membership functions define the degree of certainty (resp. possibility) that m belongs to the ensemble.
- To each disorder d is associated the twofold fuzzy set $(Md^+(m), Md^-(m)^C)$, whose respective membership functions define the degree of necessity (resp. possibility) that d alone causes m.



Design of characteristic functions

- $Md^+(m)$ is defined from the PDF of $X^m(t)$ associated with a given d at a given t.
- Without more information, $Md^-(m)^C$ is set to 1 everywhere but in regions m where no members have ever been observed (0).
- $M^+(m)$ is defined by associating a given symmetrical membership function taking value 1 in m and decreasing with distance to m.
- $M^-(m)^C$ is designed to enforce consistence with the fuzzy set $M^+(m)$.
- The fuzzy set of the potential and relevant disorders given an EPS are respectively given respectively by:



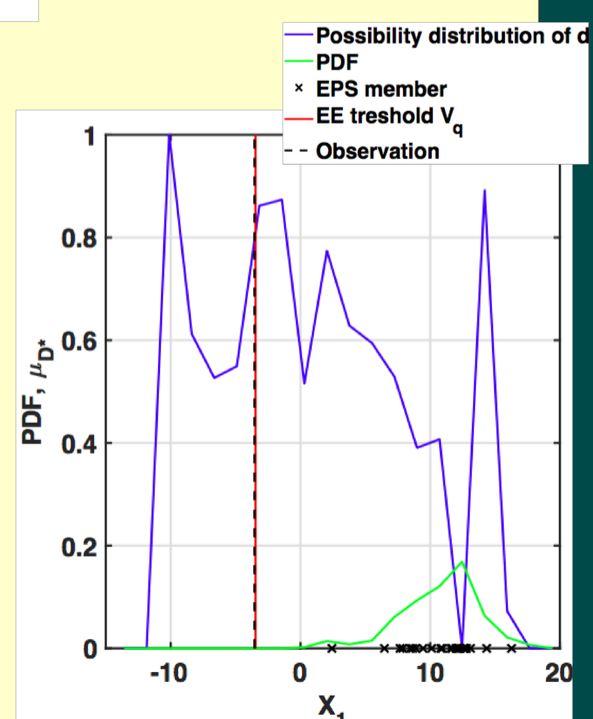
$$\hat{D}_{crisp} = \{d \in \mathcal{D}, M(d)^+ \cap M^- = \varnothing \text{ AND } M(d)^- \cap M^+ = \varnothing\}$$
$$\hat{D}^* = \{d \in \hat{D}_{crisp}, M(d)^+ \cap M^+ \neq \varnothing \text{ AND } M(d)^- \cap M^- \neq \varnothing\}$$

- Their membership function respectively read:

$$\mu_{\hat{D}} = \min\left(1 - cons(M(d)^+, M^-), 1 - cons(M(d)^-, M^+)\right)$$
$$\mu^*_{\hat{D}}(d) = \min\left(\mu_{\hat{D}}(d), \max(cons(M(d)^+, M^+), cons(M(d)^-, M^-))\right)$$

- We consider the prediction of the EE E='$X<V_q$' with V_q the quantile of interest of the climatological distribution of X. The degree of consistency of E with the resulting possibility distribution and the degree to which the later certainly implies it provide upper and lower bounds on the true probability of E [2]:

$$\min_{d \notin E} \mu^*_{\hat{D}}(d) \leq P(X < V_q) \leq \max_{d \in E} \mu^*_{\hat{D}}(d)$$



References

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