Bayesian Decisions using Regions of Practical Equivalence (ROPE) and Imprecise Loss Functions

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1 An Applied Bayesian Decision Problem

A decision between two actions a_0 and a_1 is of interest, i.e. $\mathcal{A} = \{a_0, a_1\}$.
The observed data x are modeled parametrically with parameter $\theta \in \Theta$.
The prior density $\pi(\theta)$ gets updated to the posterior density $\pi(\theta|x)$.

2 The Ideal Bayesian Solution

With the loss function $L: \Theta \times \mathcal{A} \to \mathbb{R}_0^+$,
the expected posterior loss

$$\rho(a) = \int L(\theta, a)\, \pi(\theta|x)\, d\theta \quad (\text{for } a \in \mathcal{A})$$

can be minimized to obtain the optimal action

$$a^* = \min_{a \in \mathcal{A}} \rho(a)\,.$$

Problem 3

Typically, the loss function L is inaccessible.

4 Region of Practical Equivalence (ROPE)

The action a_0 is in accordance with a parameter null value $\theta^* \in \Theta$.

A ROPE for θ^* is a "range of parameter values that are equivalent to the null value for practical purposes" [Kruschke 2018, p. 272].

Its limits "depend on the practical purpose" [Kruschke 2015, p. 338] and need to be specified reasonably.

Denote the ROPE as Θ_0 and $\Theta_1 = \Theta \backslash \Theta_0$.

The actions a_0 and a_1 are in accordance with Θ_0 and Θ_1, respectively.

Simplification

5 ROPE in Decision Theory

Practical equivalence as constant loss function:

$$\forall \theta_i, \theta_j \in \Theta_0\colon\ \forall a \in \mathcal{A}\colon\ L(\theta_i, a) = L(\theta_j, a)$$
$$\forall \theta_i, \theta_j \in \Theta_1\colon\ \forall a \in \mathcal{A}\colon\ L(\theta_i, a) = L(\theta_j, a)$$

6 Loss in Regret Form

$L(\theta, a)$	$\theta \in \Theta_0$	$\theta \in \Theta_1$
a_0	0	k_0
a_1	k_1	0

Define $k = k_1 / k_0$.

7 The ROPE-based Solution

With the ratio of expected posterior losses

$$r(k) = \frac{\rho(a_1)}{\rho(a_0)} = k\, \frac{p(\theta \in \Theta_0\, |x)}{p(\theta \in \Theta_1\, |x)}$$

the optimal action is

$$a^*(k) = \begin{cases} a_0 & \text{if } r(k) > 1 \\ a_1 & \text{if } r(k) < 1 \end{cases}.$$

Problem 8

Typically, k cannot be specified unambiguously.
Any precise value for k would be arbitrary.

Framework of Imprecise Probabilities 9

There is more to uncertainty than can be captured by precise probability values.

Sets of probabilities are employed as imprecise probabilties instead of precise probability values.

Imprecise probabilities are treated as an entity of its own.

Counteract Arbitrariness

10 An Imprecise Loss

The loss function L is specified imprecisely by $K = [\underline{K}, \overline{K}]$ instead of a precise k.
The precise values $\underline{K}$ and $\overline{K}$ are bounds for reasonable choices of k.

The Imprecise ROPE-based Solution 11

Choose the action that is optimal for all values $k \in K$:

$$a^*(K) = \begin{cases} a_0 & \text{if } r(\underline{K}) > 1 \\ a_1 & \text{if } r(\overline{K}) < 1 \end{cases}.$$

Else, withhold a decision.

An Alternative? 12

Kruschke [e.g. 2015, 2018] proposes the HDI+ROPE decision rule:

Calculate the 95% highest density interval (HDI) from the posterior density $\pi(\theta|x)$.

- If HDI is completely within the ROPE, then choose a_0.
- If HDI is completely outside the ROPE, then choose a_1.
- Else, withhold a decision.

Comparison 13

Ideal Solution	Imprecise ROPE-based Solution	HDI+ROPE decision rule
full posterior information	full posterior information	only HDI
true loss function	simplified loss function	no loss function based on a_0 and a_1
all information about loss	ROPE + loss magnitude as available	only ROPE
not arbitrary	not arbitrary	arbitrary HDI confidence level (95%)

J. Kruschke (2015). *Doing Bayesian data analysis: A tutorial with R, JAGS, and Stan.* Academic Press.

J. Kruschke (2018). Rejecting or accepting parameter values in Bayesian estimation. *Advances in Methods and Practices in Psychological Science*, 1(2):270–280.

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