Let's replace p-values with betting outcomes.

Glenn Shafer

- A 5% significance test of P is an all-or-nothing bet. You multiply your money by 0 or 20.
- A more general kind of bet is a nonnegative variable S satisfying $\mathbf{E}_P(S) = 1$. The observed value of S is your *betting score*.

$\mathbf{E}_P(S) = 1$ can be written $\sum S(y)P(y) = 1$.

So SP is a probability distribution. Call it Q.

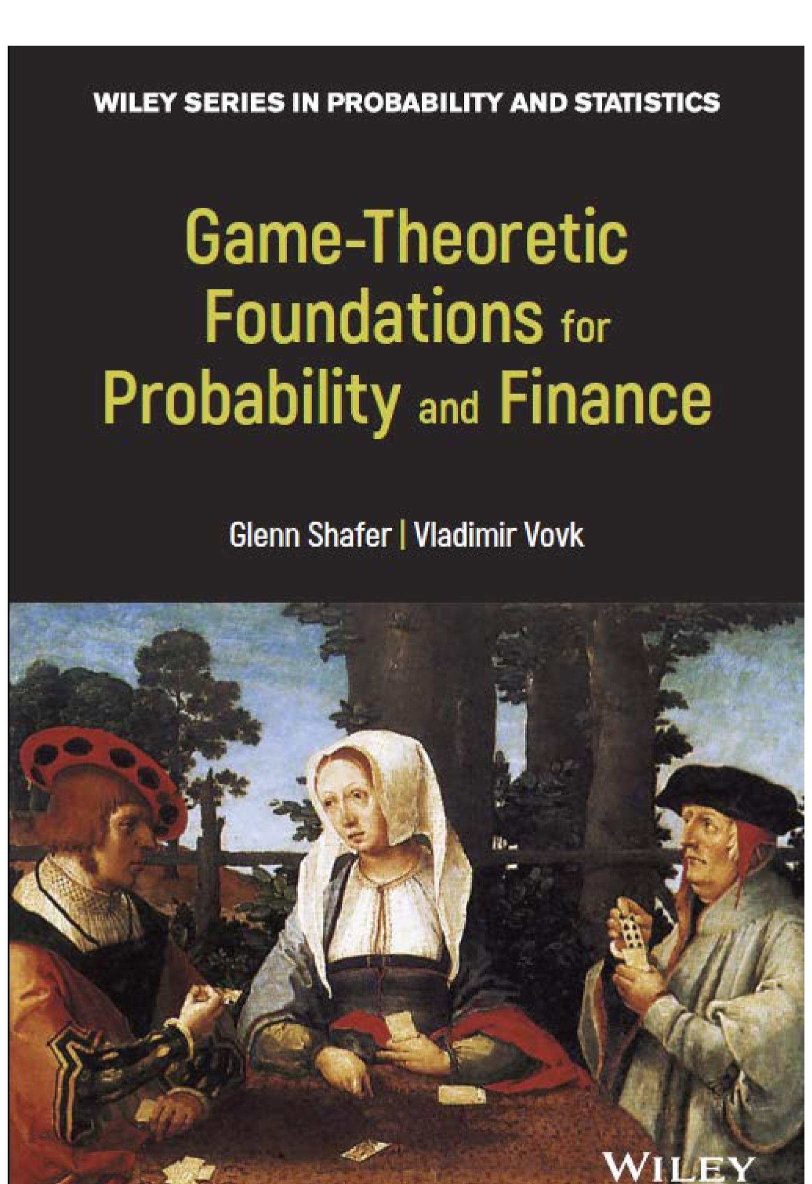
Then $S(y) = Q(y)/P(y)$.

A betting score is a likelihood ratio.

	name	notation
Proposed study		
initially unknown outcome	phenomenon	Y
probability distribution for Y	null hypothesis	P
nonnegative function of Y with expected value 1 under P	bet	S
SP	implied alternative	Q
$\exp\left(\mathbf{E}_Q(\ln S)\right)$	implied target	S^*
Results		
actual value of Y	outcome	y
factor by which money risked has been multiplied	betting score	$S(y)$

	standard concept	betting concept
strength of evidence	p-value	betting score
strength of test	power	implied target
estimation interval	$(1-\alpha)$-confidence	K-fold warranty

Kelly (1956): it is the logarithm which is additive in repeated bets and to which the law of large numbers applies.



Ever since Kolmogorov's *Grundbegriffe*, the standard mathematical treatment of probability theory has been measure-theoretic. In this ground-breaking work, Shafer and Vovk give a game-theoretic foundation instead. While being just as rigorous, the game-theoretic approach allows for vast and useful generalizations of classical measure-theoretic results, while also giving rise to new, radical ideas for prediction, statistics and mathematical finance without stochastic assumptions. The authors set out their theory in great detail, resulting in what is definitely one of the most important books on the foundations of probability to have appeared in the last few decades.

– Peter Grünwald, CWI and University of Leiden

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