The Joy of Probabilistic Answer Set Programming

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Goal: to show that

the credal semantics for Probabilistic Answer Set Programming (PASP) leads to a very useful modeling language.

Answer set programming (ASP)

- A program is a set of rules such as

$$\text{green}(X) \vee \text{green}(X) \vee \text{blue}(X) :- \text{node}(X), \textbf{not}\ \text{barred}(X).$$

- A fact is a rule with no subgoals: node(1)..
- Stable model semantics:
  - Herbrand base: all groundings generated by constants in the program.
  - Minimal model is a model (interpretation that satisfies all rules) such that none of its subsets is a model.
  - Answer set: a minimal model of the *reduct* (propositional program obtained by grounding, then removing rules with **not**, then removing negated subgoals).

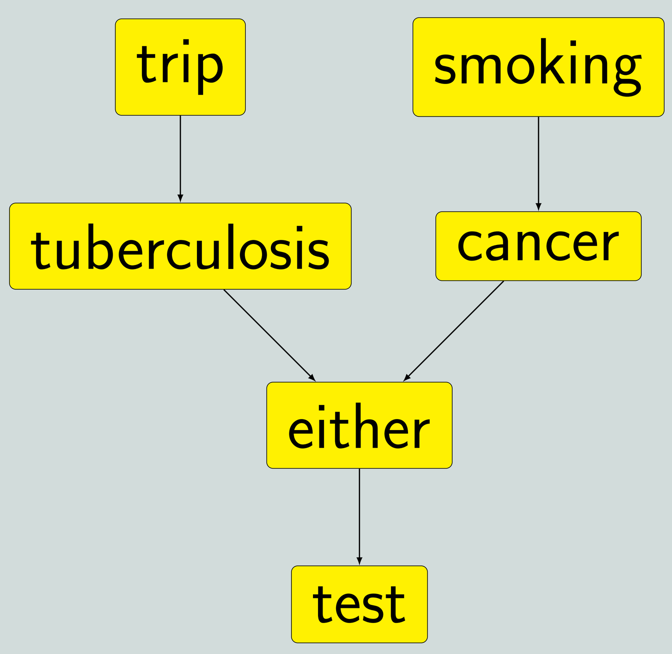
Probabilistic ASP (PASP)

- A PASP program contains rules, facts, and *probabilistic facts* such as

$$0.25 :: \text{edge}(\text{node1}, \text{node2}).$$

  A *total choice* induces an Answer Set Program.
- Acyclic propositional (Bayesian network):

0.01 :: trip.
0.5 :: smoking.
tuberculosis :− trip, a1.
tuberculosis :− **not** trip, a2.
0.05 :: a1. 0.01 :: a2.
cancer :− smoking, a3.
cancer :− **not** smoking, a4.
0.1 :: a3. 0.01 :: a4.
either :− tuberculosis.
either :− cancer.
test :− either, a5. 0.98 :: a5.
test :− either, a6. 0.05 :: a6.



- Acyclic relational:

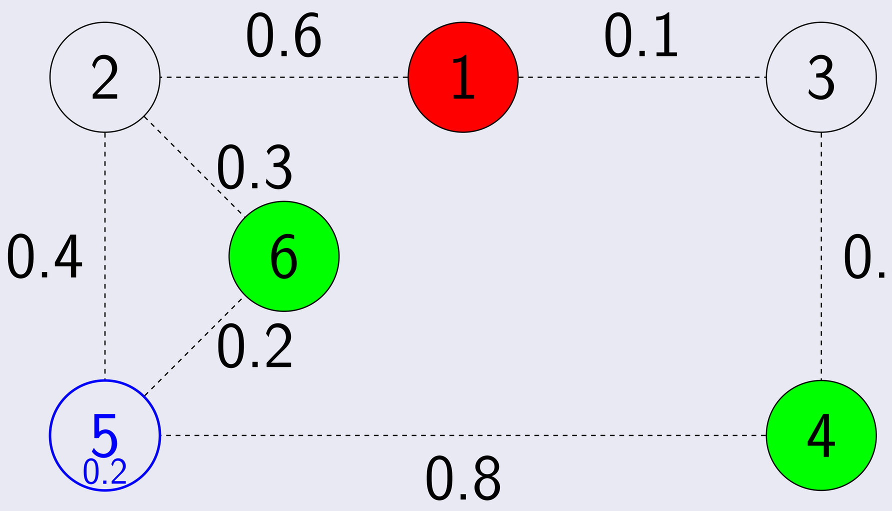
apt(X) :− student(X), a1. 0.7 :: a1.
easy(Y) :− course(X), a2. 0.4 :: a2.
pass(X, Y) :− student(X), apt(X), course(Y), easy(Y).
pass(X, Y) :− student(X), apt(X),
  course(Y), **not** easy(Y), a3. 0.8 :: a3..

This work was partially supported by CNPq and FAPESP.

Stratified programs

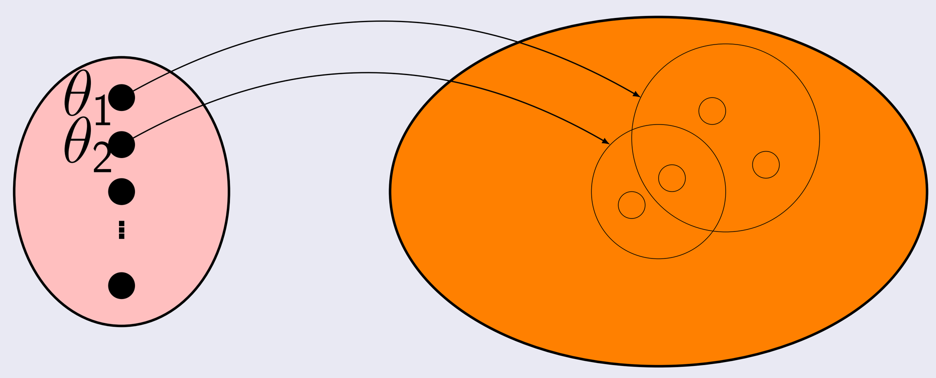
edge(X, Y) :− edge(Y, X).
path(X, Y) :− edge(X, Y).
path(X, Y) :− edge(X, Z), path(Z, Y)..

0.6 :: edge(1, 2).
0.1 :: edge(1, 3).
0.4 :: edge(2, 5).
0.3 :: edge(2, 6).
0.3 :: edge(3, 4).
0.8 :: edge(4, 5).
0.2 :: edge(5, 6).



PASP: Credal semantics

- A total choice may induce a program with many answer sets.
- Probability of each total choice may be distributed freely over answer sets: semantics is a credal set that dominates a two-monotone capacity.



- Three-coloring:

red(X) ∨ green(X) ∨ blue(X) :− node(X).
edge(X, Y) :− edge(Y, X).
¬colorable :− edge(X, Y), red(X), red(Y).
¬colorable :− edge(X, Y), green(X), green(Y).
¬colorable :− edge(X, Y), blue(X), blue(Y).
red(X) :− ¬colorable, node(X), **not** ¬red(X).
green(X) :− ¬colorable, node(X), **not** ¬green(X).
blue(X) :− ¬colorable, node(X), **not** ¬blue(X)..

Then: $\overline{\mathbb{P}}(\text{colorable}, \text{blue}(3)) = 0.976$.

- Lower/upper probabilities: *sharp* probabilities with respect to appropriate questions: "What is the probability that I will be able to select a three-ordering where node 2 is red?" — answer is $\overline{\mathbb{P}}(\text{colorable}, \text{red}(2))$.

Closing...

- In the paper: algorithm to compute lower/upper probabilities (future: better algorithms...).
- In short: PASP with credal semantics is a very powerful language.
  - We can compute probabilities with some implicit quantification.