Confidence in belief, weight of evidence and uncertainty reporting

Brian Hill

GREGHEC, CNRS & HEC Paris

Introduction

Keynes / Ellsberg Example

Unknown urn: 100 balls, each red or black (no more information).

Known urn: 100 balls, 50 red, 50 black.

Keynes Your beliefs about the colour of the next ball drawn?

- **Balance of evidence**: same
- **Weight of evidence**: different
- Bayesian belief: same ($\frac{1}{2}$).

Ellsberg Which urn would you rather bet on?

- Most people: Known urn
- Bayesian decision: indifferent.

These preferences can be justified by:

- a higher **weight of evidence** in the case of the known urn
- higher **confidence in one's belief / credal judgement** of $\frac{1}{2}$ for red on the next draw from the known urn

Bayesianism denies **weight of evidence** and **confidence in beliefs** any role in choice.

But Uncertainty reporting in the form of:

- Beliefs / credal judgements / probability assessments
- Confidence in them

is used by the **IPCC**, **US Defense Intelligence Agency**.

Objectives

1. A model of **confidence in belief / weight of evidence**
2. A framework for **effective uncertainty reporting**

Desiderata for 2. (in theory):

a. **Clean belief / value separation**

b. **Uncertainty language supporting unambiguous inter-agent communication**

A. The larger project

Develop an account of confidence in beliefs that:

✓ Includes a model of (confidence in) beliefs

✓ Brings out its role in (rational) decision making

✓ Defends its normative credentials

✓ Sheds lights on its role in communicating severe uncertainty (e.g. IPCC)

✓ Incorporates a theory of belief updating

- and techniques for confidence elicitation ...

1. Model

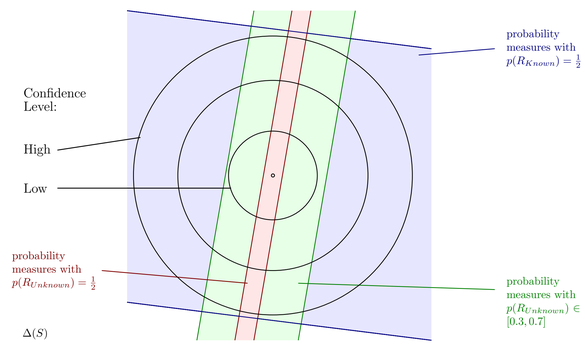

Figure 1: Confidence Ranking

Confidence Ranking: nested family of sets of probability measures (i.e. of subsets of $\Delta(S)$). [2]

- Each set: confidence level
- Larger sets: higher confidence
- Credal statements (e.g. $p(R) \geq 0.3$) holding everywhere in a set: those judgements held at that level of confidence.

This model:

- Confidence / weight of evidence **admits of degrees**
- It is **ordinal**
- Confidence rankings **portray** the precision / weight trade-off
- ... **without asking the agent to settle it**.

2.a. Belief / Value Separation

Under IP

No clean separation between beliefs and uncertainty attitiude (see comparative statics results [3]):

- In the context of decision, sets of probability measures can reflect **both** beliefs and uncertainty tolerance / prudence.

Elicitation of probability intervals:

- reporting one interval requires trading off its accuracy against its informativeness
- ... and that needs value judgements.

Under Confidence Approach

Clean separation (see [4, 5]) between:

- **beliefs and confidence in beliefs** represented by the confidence ranking
- **attitudes to uncertainty / limited confidence** represented by the cautiousness coefficient (see box B).

Accuracy / informativeness trade-off:

- portrayed by the confidence ranking
- made, for a given decision, by the cautiousness coefficient.

2.b. Uncertainty Language

Challenge: to **calibrate** confidence levels across agents.

Calibrating Probability

How are rational agents' probability assessments calibrated?

- On "objectively uncertain / chance" events.

- $E_{0.25}$: red ball drawn from urn with 5 red / 15 blue balls.
- $E_{0.2}$: red ball drawn from urn with 4 red / 16 blue balls.

$E_{0.25}$ has an "objectively higher chance" than $E_{0.2}$.

Principal Principle If the only available information is that E_x has a higher chance than E_y, then a rational agent considers E_x to be subjectively more likely than E_y. [6]

$\Longrightarrow$ Events E_x **calibrate probability levels** across (rational) agents.
(I.e. They support an inter-agent comparable assignment of subjective probability / degree-of-belief numbers.)

Calibrating Confidence

Idea Use "objective" weight-of-evidence comparisons.

- E_n: next ball drawn from urn is red, where:
  - Urn contains 20 balls (red or blue)
  - $2n$ draws observed, n of which are red
- P_n: judgement that the probability of $E_n \geq 0.4$

Weight of evidence for P_{50} is "objectively" higher than for P_5

Weight-of-Evidence Principal Principle If the only information available supports P_m objectively with more weight of evidence than P_n, then a rational agent holds P_m with more confidence than P_n.

$\Longrightarrow$ Probability judgements P_n **calibrate confidence levels** across (rational) agents.

B. Confidence and decision

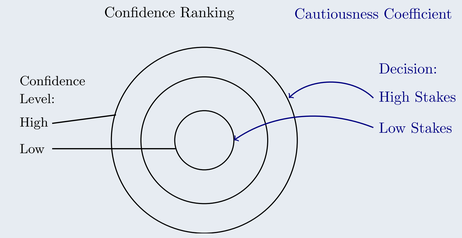

Figure 2: Confidence & Decision Making

Cautiousness coefficient D: function assigning a set in the confidence ranking to each decision [4, 5].

- Family of decision rules, each using the set identified by D.
- E.g. evaluate an act f according to $\inf_{p \in D(f)} EU_p f$ [4]

Characteristics of the family [2]:

- Faithful to the pre-formal **maxim**:

  the higher the stakes involved in the decision, the more confidence is required in a belief for it to play a role.

- **Normatively reasonable implications for choice**
  - Point of divergence from IP: weakens Stakes Independence (betting quotients are independent of the stakes).

Information / Details

Confidence Elicitation Web tool:

- `http://confidence.hec.fr/app/`.

Papers, slides, talks:

- `http://www.hec.fr/hill`
- `http://www.desevun.org`

Contact:

- hill@hec.fr

References

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[2] Brian Hill. Confidence in Beliefs and Rational Decision Making. *Economics & Philosophy*, 35(2):223–258, July 2019.

[3] Ithzak Gilboa and Massimo Marinacci. Ambiguity and the Bayesian Paradigm. In *Advances in Economics and Econometrics: Theory and Applications, Tenth World Congress of the Econometric Society*. 2013.

[4] Brian Hill. Confidence and decision. *Games and Economic Behavior*, 82:675–692, November 2013.

[5] Brian Hill. Incomplete preferences and confidence. *Journal of Mathematical Economics*, 65:83–103, August 2016.

[6] David K Lewis. A Subjectivist's Guide to Objective Chance. In *Philosophical Papers*, volume 2, pages 83–132. OUP, Oxford, 1986.