Embedding Probabilities, Utilities and Decisions in a Generalization of Abstract Dialectical Frameworks

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Introduction

- Life is made up of long list of decisions, from choosing a healthy lunch to choosing a profession, affected by uncertainties and preferences.
  - The uncertainty mostly arises because of external factors, called **states**, out of control of agents; uncertainties can be modeled by probabilities.
  - An agent usually knows the set of possible **outcomes** of a decision and has a preferences on them; preferences can be modeled by utilities.
- **Expected Utility** deals with problems in which probabilities of states and utilities of outcomes play a role in the choice.
- **Argumentation theory** can shed light on the process of decision making, from modeling to evaluating a problem.
- Main goal is to propose an argumentation formalism, ***numerical abstract dialectical frameworks*** (nADFs) that can model decision problems.

Decisions and Argumentation

Decision Problem

- A **decision problem** is a tuple (A, S, O, p, u) where:
  - A is a finite set of actions;
  - S is a finite set of states;
  - O is a finite set of outcomes;
  - p is a probability function on states, $p : S \to [0, 1]$ s.t. $\Sigma_s p(s) = 1$;
  - u is a utility function on outcomes, $u : O \to [0, 1] \cap \mathbb{Q}$.
- The **expected utility** of $a \in A$ is defined as:
  $$EU(a) = \Sigma_{o \in O} p(s|a, o)u(o)$$
- **Maximum expected utility**(MEU), $a \in$ MEU if for each $a' \in A$, $EU(a) \geq EU(a')$.

Argumentation Formalism

- An **abstract dialectical framework** (ADF) is a tuple $D = (N, L, C)$ where:
  - N is a finite set of nodes;
  - $L \subseteq N \times N$ is a set of links;
  - $C = \{C_n\}_{n \in N}$ is a collection of total functions $C_n : (par(n) \to \{\mathbf{t}, \mathbf{f}\}) \to \{\mathbf{t}, \mathbf{f}\}$.
- A **three-valued interpretation**: $v : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$.
- The **information ordering** $\leq_i$: $\mathbf{u} \leq_i \mathbf{t}$ and $\mathbf{u} \leq_i \mathbf{f}$.
- $v_i \leq_i v_j$ iff $\forall a \in A : v_i(a) \leq_i v_j(a)$.
- $[v]_c = \{w \in \mathcal{V}_c \mid v \leq_i w\}$
- $\Gamma_F(v)(n) = \bigsqcap\{C_n(w) \mid w \in [v]_c\}$
- Semantics of ADFs:
  - $v \in adm(F)$ if $v \leq_i \Gamma_F(v)$
  - $v \in pref(F)$ if v is $\leq_i$-maximal admissible
  - $v \in comp(F)$ if $v = \Gamma_F(v)$
  - v is $grd(F)$ if v is the $\leq_i$-least fixed point of $\Gamma_F(v)$
  - $v \in mod(F)$ if v is a two-valued interpretation and $v = \Gamma_F(v)$

Numerical Abstract Dialectical Frameworks

- nADFs enhance ADFs by allowing numerical acceptance conditions of arguments and arithmetical computations among them.
- The logic used in nADFs is a variation of propositional logic, consists of:
  - binary function symbols: $\oplus$ and $\otimes$;
  - a binary predicate symbol: $\succeq$
- Let V be $[0, 1] \cap \mathbb{Q}$. An **nADF** is a tuple $U = (N, L, C, i)$
  - N is a finite set of nodes;
  - $L \subseteq N \times N$ is a set of links;
  - $C = \{C_n\}_{n \in N}$, $C_n : (par(n) \to V) \to V$;
  - i is an input function , $i : N' \to V$ where $N' \subseteq N$.
- A **many-valued interpretation**: $v : N \to V_{\mathbf{u}}$, $V_{\mathbf{u}} = ([0, 1] \cap \mathbb{Q}) \cup \{\mathbf{u}\}$
- i**-correction** of v: $\mathbf{v}(n) = \begin{cases} i(n) & \text{if } i \text{ is defined on } n, \\ v(n) & \text{otherwise.} \end{cases}$
- The evaluation of non-standard connectives:
  - $\mathbf{v}(A \wedge B) := \min\{\mathbf{v}(A), \mathbf{v}(B)\}$
  - $\mathbf{v}(A \vee B) := \max\{\mathbf{v}(A), \mathbf{v}(B)\}$
  - $\mathbf{v}(a \otimes b) := \mathbf{v}(a) \times \mathbf{v}(b)$
  - $\mathbf{v}(a \oplus b) := \mathbf{v}(a) + \mathbf{v}(b)$
  - $\mathbf{v}(t_1 \succeq t_2) := \begin{cases} 1 & \text{if } \mathbf{v}(t_1), \mathbf{v}(t_2) \in \mathbb{Q} \text{ and } \mathbf{v}(t_1) \geq \mathbf{v}(t_2), \\ 0 & \text{if } \mathbf{v}(t_1), \mathbf{v}(t_2) \in \mathbb{Q} \text{ and } \mathbf{v}(t_1) < \mathbf{v}(t_2), \\ \mathbf{u} & \text{if either } \mathbf{v}(t_1) \text{ or } \mathbf{v}(t_2) \text{ is undecided.} \end{cases}$

Embedding of Decision Problems in nADFs

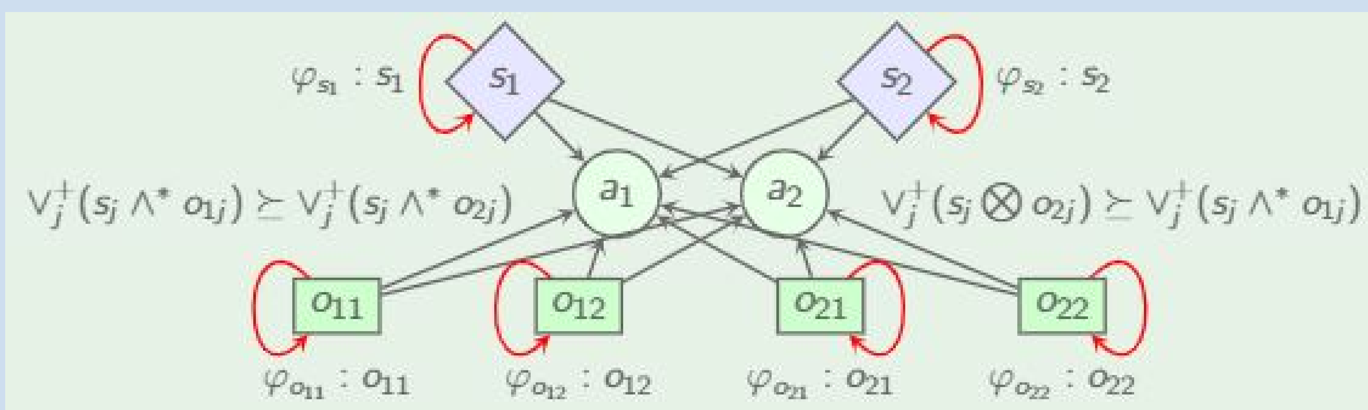
A decision problem $D = (A, S, O, p, u)$ can be modeled by nADF $U_D = (N, L, C, i)$ as follows:

- $N = A \cup S \cup O$;
- $\varphi_s = s$ for $s \in S$;
  $\varphi_o = o$ for $o \in O$;
  $\varphi_{a_i} = \bigotimes_{i \neq k}(\bigoplus_j (s_j \otimes o_{ij}) \succeq \bigoplus_j (s_j \otimes o_{kj}))$ for $a_i \in A$;
- $i(s) = p(s)$ for $s \in S$ and $i(o) = u(o)$ for $o \in O$.

Example

a_1/a_2: whether or not to buy an international insurance for 100 euros.

- o_{11} buying and needing
- o_{12} buying and not needing
- o_{21} not buying but needing
- o_{22} not buying and not needing



Overview of results

Let $D = (A, S, O, p, u)$ be a decision problem, let $U_D = (N, L, C, i)$ be the corresponding nADF.
Theorem: All semantics of U_D coincide.
Theorem: Let v be the grounded interpretation of U_D.
The set A_1^v equals the set of actions with maximal expected utility in the decision problem D.