Decisions & Algorithms: How To Get Your Act Together?

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6 July, 2019

Aim Of This Talk

- share my views on why I think this topic is important
- share my views on the past, present, and future
  of decision making under severe uncertainty & related algorithms
- generate discussion about imprecise probability
- above all, to inspire you

Outline

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Severe Uncertainty & Statistics

- what is your first response to situations that involve severe uncertainty?
- statistics = a method for 'objectively'
  - quantifying what we do know, acknowledging limitations
  - make recommendations e.g. in the form of decisions
- this goes back to the very beginnings of human culture
  - e.g. uncertainty about survival, harvest, ... $\rightarrow$ ritual sacrifice
  - we use other methods these days
- what you deem 'objective' depends on your core belief system

Severe Uncertainty & Statistics

statistician at work for fortune and glory
(disclaimer: may not be an entirely historically correct depiction)

Why Care Today?

Why do I care

- mankind faces huge challenges
  - climate change
  - decline in biodiversity
  - drastic changes to environment, impact hard to model
  - culture of dismissal of scientific evidence
- how to properly acknowledge and communicate severe uncertainty?
- how can decision makers act in face of these issues?

Legal requirements: "When an activity raises threats of harm to human health or the environment, precautionary measures should be taken even if some cause and effect relationships are not fully established scientifically."

imprecise probability can send a positive message
in its ability to help addressing these problems!

Why Care Today?

Imprecise Probability = Natural Model For

- elicitation of severe uncertainty: desirability
- applying the precautionary principle: Γ-maximin and its friends

Implications

- we can reach out as a community to sell this message
- what can imprecise probability say about communication of severe uncertainty?
- there's a need to make decision making based on imprecise probability practical

Outline

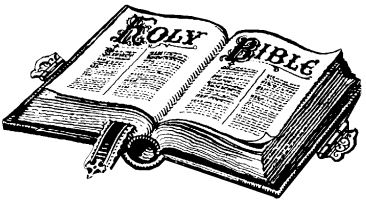
Etymology of Probability

until about the 18th century

Definition

probable = approvable by opinion
(not by evidential support!)

- of respected people
- of ancient books



the earth is probably flat

epistemic flavour!

Probabilism in Religion

- one is to follow 'probable' sources
- what if sources contradict one another?

Casuists

Pick *any* probable source (whatever suits you best!).

Jansenists

First consider moral and social consequences,
then find an old text which approves.

Blaise Pascal was a Jansenist.

https://en.wikipedia.org/wiki/Jansenism

Pascal: The First Imprecise Probabilist?

(analysis based on Hacking [2])

- partitioning of the possibility space
  - ω_1 there is a God as depicted by the church
  - ω_2 there is no God
- actions
  - d_1 take God into account with everything you do
  - d_2 be entirely indifferent as to whether God exists or not
- which action to take?
- (note: Pascal doesn't try answering whether God exists or not!! He's asking whether you should behave as if God exists or not!)

Pascal's Wager: Solution I

- for some $\alpha > 0$ and $\beta > 0$:

	ω_1	ω_2
d_1	α	0
d_2	$-\beta$	0

- dominance: take d_1
- criticism: are you sure there is no difference in utility for taking d_1 or d_2 if ω_2 obtains?

Pascal's Wager: Solution II

- for some $\alpha > \gamma > 0$ and $\beta > 0$

	ω_1	ω_2
d_1	α	$-\gamma$
d_2	$-\beta$	0
probability	$1/2$	$1/2$

- maximize expectation: d_1 has higher expectation than d_2, so take d_1
- criticism: you don't know the probabilities of ω_1 and ω_2

Pascal's Wager: Solution III

- for some $\gamma > 0$ and $\beta > 0$

	ω_1	ω_2
d_1	$+\infty$	$-\gamma$
d_2	$-\beta$	0
probability	p	$1-p$

- dominating expectation: for every value of p (strictly positive, but no matter how small) d_1 has higher expectation than d_2, so take d_1
- this directly links to imprecise probability theory (E-admissibility)

Laplace

- first modern axiomatic foundation of probability theory [3]
- first unambiguous statement of the principle of indifference:
  adoption of the uniform prior as a model for complete ignorance (severe uncertainty!)

Boole: The Godfather of Imprecise Probability

1854: critique on Laplace's treatment of probability [1]

- requirement of completeness is too strong
  we may not always be able to handle *any* system of probabilities; e.g.

$$P(A) = P(A|B_1)P(B_1) + \cdots + P(A|B_k)P(B_k) \tag{1}$$

  do we always find a partition
  in which all terms on the right hand side are perfectly known?
- prior ignorance is not properly handled by the uniform distribution
- Boole suggests a probability calculus based on bounding (lower and upper probability)

Outline

Recent Developments

decade	contribution	details
1970's	desirability	puts decision making at the forefront of uncertainty modelling
1980's	imprecise decision rules	how to use imprecise probability/utility in decision problems
1990's	linear programming	recognized as an algorithm for solving decision problems with imprecise probability
	decision axioms	extension of Anscombe/Aumann
	issues with extensive-normal form equivalence	'paradoxes' with sequential decision making
	SIPTA is created	imprecise probability becomes widely recognized

Recent Developments

decade	**contribution**	**details**
2000's	credal classification	machine learning, practical applications
	special purpose linear programming formulations	for applying specific decision rules
	optimal control with imprecision	algorithms for dynamic programming, practical applications
	decision trees with imprecision	algorithms for sequential decision making, practical applications

Recent Developments

decade	**contribution**	**details**
2010's	issues with extensive-normal form equivalence	'consistent' sequential decision making prohibits imprecision...
	new alternative decision rules	beyond maximality, E-admissibility, interval dominance, Γ-maximin and its friends
	enhanced algorithms & linear programming methods	custom built for imprecise decision rules, new sampling methods
	full axiomatic system	covers most decision rules

Outline

Open Problems

Act-State Dependence

- when actions affect (part of) the state of the world, not just our utility
- essential in most environmental decision problems!
- possible to write down 'reasonable' decision rules (based on interval dominance)
- axiomatisation of act-state dependent decision rules = open problem

Open Problems

Machine Learning

sequential decision problems where dynamics and reward structure are initially unknown

- exploration/exploitation trade-off
- inherently opportunistic
- precautionary approaches to machine learning = largely uncharted territory
- (e.g. imprecise temporal difference learning?)

Open Problems

Communication

how do we make imprecise probability part of the standard decision making repertoire for:

- scientific consultation
- industrial planning
- government policy making
- ...

Thank you for listening!

Questions?

Cheers to the next 20 years of SIPTA with many more exciting things to come!

[1] George Boole.
An investigation of the laws of thought on which are founded the mathematical theories of logic and probabilities.
Walton and Maberly, London, 1854.

[2] Ian Hacking.
The Emergence of Probability: A Philosophical Study of Early Ideas about Probability, Induction and Statistical Inference.
Cambridge University Press, 1975.

[3] Pierre Simon Laplace.
Essai philosophique sur les probabilitiés.
Bachelier, Paris, 1825.
Cinquième édition.