

Natural Selection with Objective Imprecise Probability

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ISIPTA 2019
July 5, 2018

Evolutionary assumptions

- Natural selection occurs in a populations of organisms when differences in biological fitness between heritable traits cause changes in relative frequencies of organisms with those traits.
- Fitness differences depend on (objective) probabilities of outcomes, such as organisms with a trait having particular numbers of offspring.
- Fitness involves tradeoffs. e.g. if a bird's body uses carotenoids for feather coloring that attracts mates, there will be less of these substances available for responding to parasites that attack the birds.

Probabilistic assumptions

- Objective probabilities over a space of specified outcomes are realized by repeatable “setups” such as *pair of dice of uniform density being shaken vigorously and tossed by a person*.
- Objective probabilities relative to a setup are consistent with underlying determinism in particular instances of the setup.
- Not all setups realize probabilities. For some, the outcomes may be erratic—i.e. they have no probabilities relative to the setup—or imprecisely probabilistic.
- Average ink percentage of paper in pockets of people looking at a poster while a bicyclist wearing green rides past one kilometer to the west.

Natural selection with imprecise probabilities?

- Fitnesses of traits in a population depend on the characteristics of the environment.
- An environment can include states that vary probabilistically. Which state occurs can affect the probabilities of outcomes that fitness depends on.
- Overall fitness in an environment is a probability-weighted average of fitnesses in different “subenvironments”.
- What if environments vary erratically?

Main argument

1. Natural selection sometimes produces patterns of behaviors in members of species S_1 that are imprecisely probabilistically distributed, conditional on perceived environmental conditions: Precisely calibrated, probabilistic behaviors are too costly.
2. These behaviors form part of the environment for members of another species S_2 (predators, prey, competitors, disease vectors, etc.).
3. So the S_2 population's environment includes imprecisely probabilistic conditions that can affect success in producing descendants.

Thus natural selection often depends on objective imprecise probabilities.

Argument for premise 1

1. Let environmental states have precise objective probabilities.
2. Natural selection should favor traits producing optimal behaviors conditional on perceptions of environmental state.
3. Behavior narrowly distributed around an optimum is expensive: Nervous systems, muscles, bone, etc. require time to build, and energy to maintain.
4. Probabilistic behavior with somewhat miscalibrated mean or other parameters is less expensive, and might be good enough—i.e. better than competitors.
5. Imprecisely probabilistic behavior should be even less expensive, and could be good enough in the same sense.

This is why natural selection sometimes produces patterns of behaviors in members of species S_1 that are imprecisely probabilistically distributed, conditional on perceived environmental conditions.

Imprecise fitness and choice functions

- Simplest fitness measure $w_e(x)$ is the expected number of offspring for a trait x in environment e .
- If subenvironments vary erratically: lower/upper (objective) previsions, infimum $\underline{w}(x)$, supremum $\overline{w}(x)$ of precise fitnesses in subenvironments.
- Trait A_1 is fitter than trait A_2 if A_1 interval dominates A_2 :
 $A_1 \supset A_2$ iff $\underline{w}(A_1) > \overline{w}(A_2)$.
- Trait A_1 is fitter than trait A_2 if environments vary erratically so that the entire population experiences the same environment at t , and A_1 dominates across population-wide environments:
Then A_1 is fitter_{dp} than A_2 iff $(\forall e) w_e(A_1) > w_e(A_2)$.
- Other choice functions don't seem relevant. e.g. E-admissible traits won't necessarily be selected for. These are traits such that there is some particular environment that makes all of them at least as fit as all other traits: $\{A_i : (\exists e)(\forall A_j) E_e(A_i) \geq E_e(A_j)\}$.

(Precise) Wright-Fisher model

Simple Markov model of change in allele frequencies in a population of fixed size N :

(Precise) probability of transition from i to j A alleles:

$$p_{ij} = \binom{2N}{j} \eta_i^j (1 - \eta_i)^{2N-j},$$

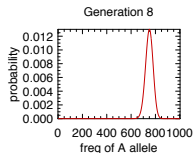
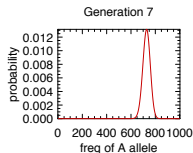
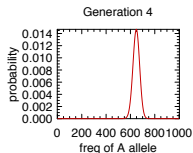
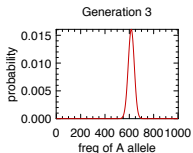
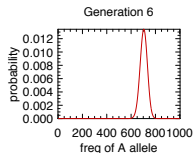
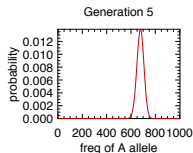
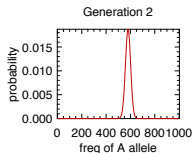
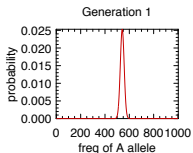
where the (precise) probability of an A allele being chosen is:

$$\eta_i = \frac{w_{AA}i^2 + w_{AB}i(2N - i)}{w_{AA}i^2 + 2w_{AB}i(2N - i) + w_{BB}(2N - i)^2}.$$

$w_{\alpha\beta}$ is the fitness of an organism with alleles (genes) α and β at the same locus (location) on two chromosomes.

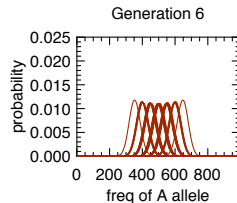
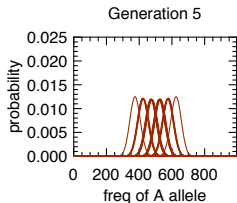
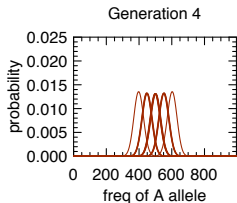
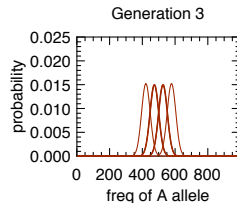
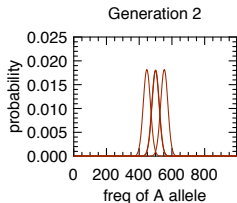
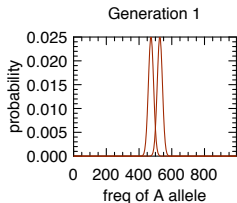
(Precise) Wright-Fisher model

Example: A is fitter than B ($w_{AA} = 1.0$, $w_{AB} = 0.95$, $w_{BB} = 0.7$):



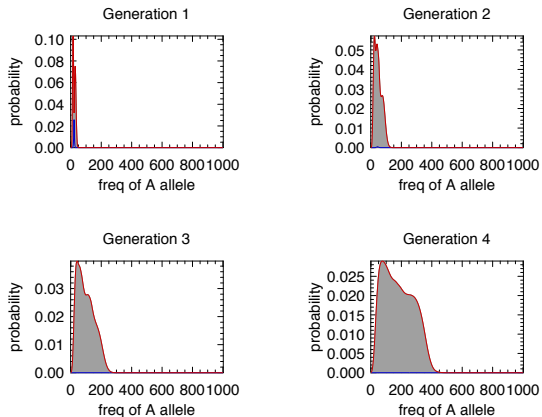
Imprecise Wright-Fisher model

Two erratically varying population-wide environments:



Imprecise Wright-Fisher model

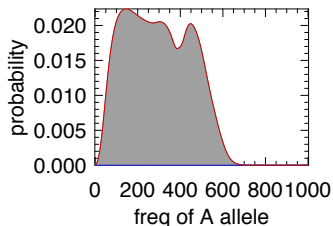
Bounds on lower/upper probabilities for frequencies of A allele with erratically determined environments, using Hartfiel's hi-lo algorithm for matrix intervals:



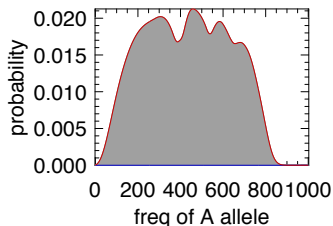
$$w_{AA} = 1.0, w_{AB} = 0.9, w_{BB} = 0.3; w_{AA} = 1.0, w_{AB} = 0.3, w_{BB} = 0.2.$$

Imprecise Wright-Fisher model

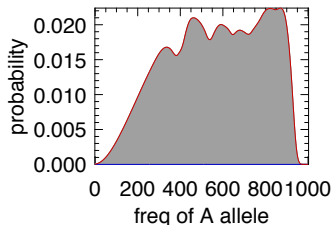
Generation 5



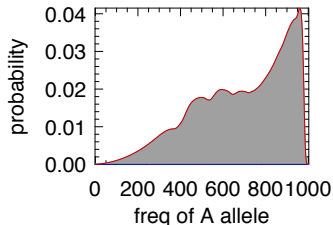
Generation 6



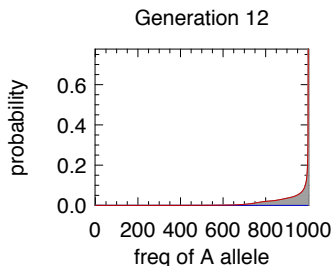
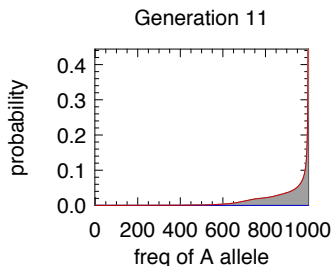
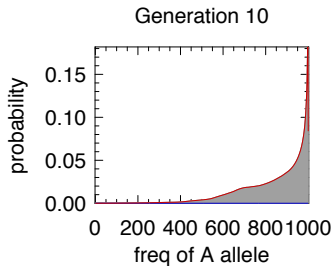
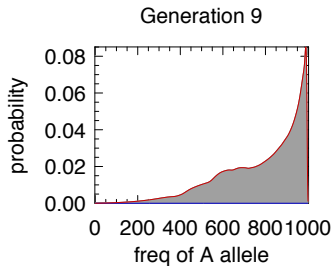
Generation 7



Generation 8



Imprecise Wright-Fisher model



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2. These behaviors form part of the environment for members of another species S_2 (predators, prey, competitors, disease vectors, etc.).
3. So the S_2 population's environment includes imprecisely probabilistic conditions that can affect success in producing descendants.
4. S_2 is part of the environment of S_1, S_3 , etc.
5. Thus natural selection often depends on objective imprecise probabilities.

Causal probability and erraticity

- *Causal probability*: Objective probability realized by a set of conditions, a *chance setup* (person tossing dice) producing outcomes, where manipulating some of these conditions (densities in the dice) manipulates probability and, usually, relative frequencies.
- I assume there are ways for causal probability to be realized by underlying deterministic dynamics, as is in dice tossing.
- *Erratic* setups have outcomes but don't realize probability of any kind, at the level of the setup. (What's the objective probability that the percentage of ink in pieces of paper in pockets of the next ten people who attend a talk at ISIPTA lies within such and such bounds?)
- Natural selection depends on probabilities of survival and reproduction for organisms with different traits *in an environment*

Imprecise fitness and decision rules

- Trait d : dig deep burrows, fitter in dry periods
Trait s : dig shallow burrows: fitter in wet periods
- Fitness $w(x)$ for $x = d, s$ in environments e : $w(x) = E_e w_e(x)$.
- If environments vary erratically: lower/upper (objective) previsions, infimum $\underline{w}(x)$, supremum $\overline{w}(x)$ precise fitnesses.
- Trait A_1 is fitter than trait A_2 if A_1 interval dominates A_2 : $A_1 \sqsupset A_2$ iff $\underline{w}(A_1) > \overline{w}(A_2)$.
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