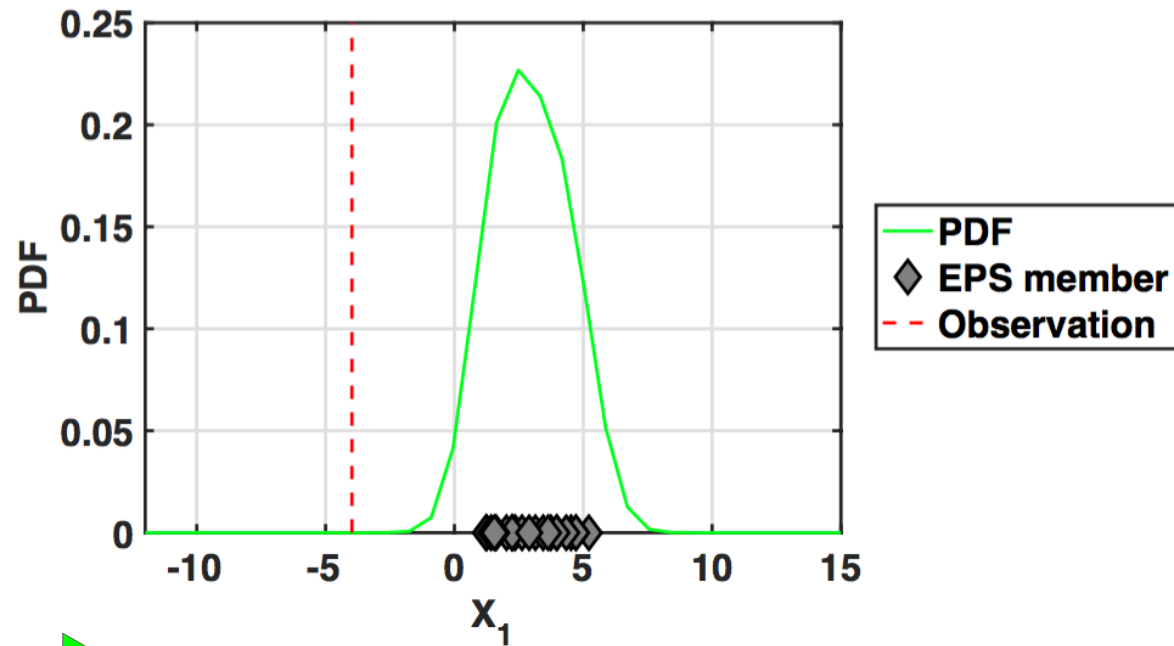
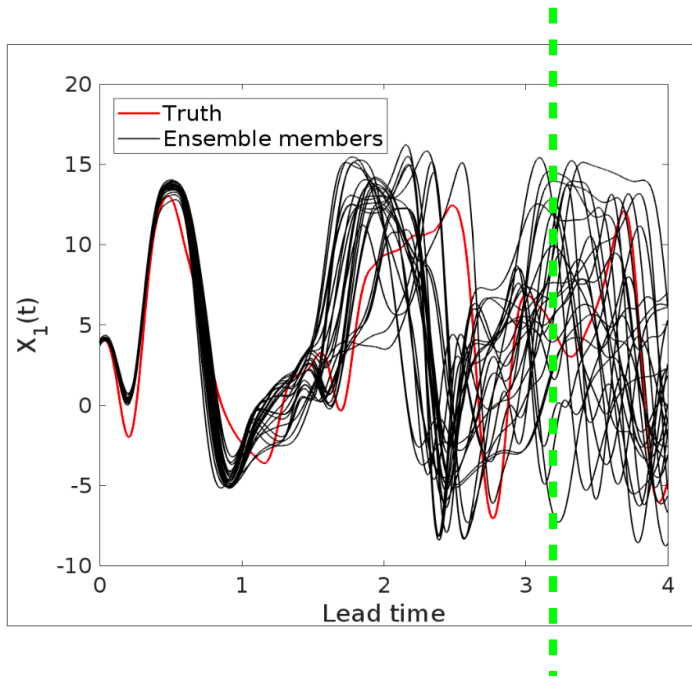


# A **possibilistic** interpretation of **ensemble predictions**: Experiments on the **imperfect** Lorenz 96 model

N. Le Carrer, S. Ferson

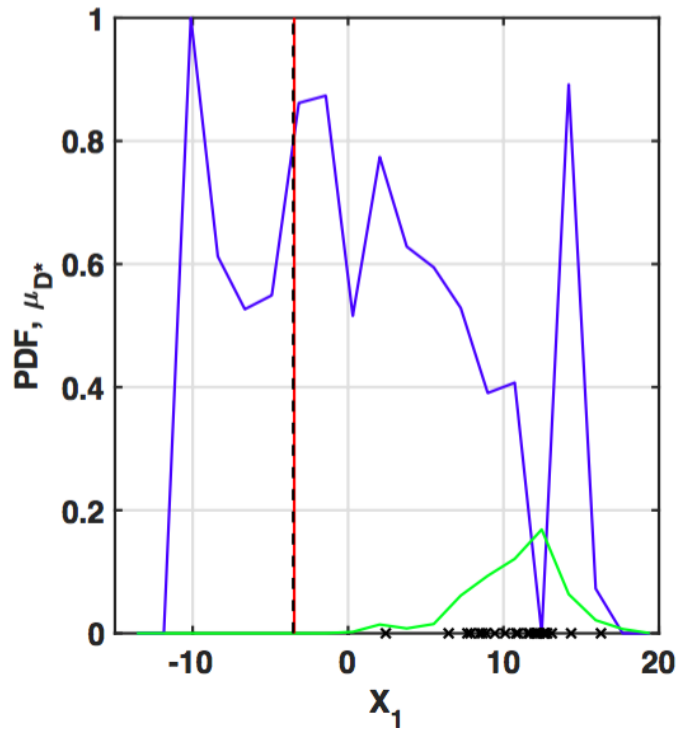
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Standard probabilistic  
interpretation  
+  
statistical post-processing



# A possibilistic interpretation?



— Possibility distribution of  $d$   
— PDF  
x EPS member  
- - EE threshold  $V_q$   
- - Observation

- Under model error and 'biased' sampling, it makes more physical sense and offers theoretical guarantees.
- Used to bound the probability of a given weather event.

## A possibilistic interpretation of ensemble predictions: Experiments on the imperfect Lorenz 96 model

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### 1. Motivations

- Ensemble weather predictions assume that the model error is dominated by initial condition (IC) error, hence a Monte-Carlo like sampling of ICs that are then run forward through the model. This assumption is shown not to be true in practice. A PDF estimated from ensemble members (EMs) shows more about the behaviour of the model than about the real system.
- The extremely high dimensionality of the weather phase space makes it in practice impossible to sample randomly ICs: methods selecting the fastest growing perturbations are used instead, to assess 'all' possible scenarios.
- Extreme events (EE) generally result from nonlinear interactions at small scale, which makes them hardly obvious in a probabilistic interpretation of ensemble forecasts.
- The probabilistic interpretation of ensemble predictions consequently generally does not work well without statistical post-processing [4].
- It consists most often in fitting local PDFs modelling the uncertainty on each member, and summing all of them to get a global PDF, supposed to estimate the location of the true system in the phase space (Bayesian model averaging, Best member dressing); or in assuming a parametric form for the global PDF and deriving its parameters from linear combinations of the ensemble's characteristics (mean and variance), e.g. Non-homogeneous Gaussian regression.
- Post-processing improves the ensemble skills for common events and extends the skillful prediction horizon. However it is shown to deteriorate significantly performances for EE.

### 2. Problem & Approach

- A probabilistic approach of mono-model ensemble prediction systems (EPS) fails to predict events that are not associated with a substantial density of EMs, which is often the case with EEs.
- We need something transcending the possibility of having the system in other areas than the one actually identified by EMs, and yet acknowledging higher probabilities resulting from local agglomeration of EMs: possibility theory, with the combination of dual possibility/necessity functions seems appropriate.
- Contrary to the current probabilistic interpretation (under model error, and biased sampling), a possibilistic development makes more physical sense and offers theoretical guarantees.
- We use it for bounding the probability of a given weather event (here EE).

### 3. Methodology

- We use the possibilistic FMECA (fault mode effect analysis) presented in [1]. The EMs  $X^m(t)$ ,  $m=1..M$  are manifestations of a disorder  $X(t)$ , that is the true system state at time  $t$ .
- Each manifestation  $m$  is characterized via the twofold fuzzy set  $(M^m(m), M(m)^F)$ , whose respective membership functions define the degree of certainty (resp. possibility) that  $m$  belongs to the ensemble.
- To each disorder  $d$  is associated the twofold fuzzy set  $(Md^m(m), Md(m)^F)$ , whose respective membership functions define the degree of necessity (resp. possibility) that  $d$  alone causes  $m$ .

Design of characteristic functions

- $Md^m(m)$  is defined from the PDF of  $X^m(t)$  associated with a given  $d$  at a given  $t$ .
- Without more information,  $Md(m)^F$  is set to 1 everywhere but in regions  $m$  where no members have ever been observed [2].
- $M^m(m)$  is defined by associating a given symmetrical membership function taking value 1 in  $m$  and decreasing with distance to  $m$ .
- $M(m)^F$  is designed to enforce consistency with the fuzzy set  $M^m(m)$ .
- The fuzzy set of the potential and relevant disorders given an EPS are respectively given by:
 
$$\tilde{D}_{lower} = \{d \in D, Md^m(m) \cap M^m = \emptyset \text{ AND } Md(m)^F \cap M^m = \emptyset\}$$

$$\tilde{D} = \{d \in D, Md^m(m) \cap M^m \neq \emptyset \text{ AND } Md(m)^F \cap M^m \neq \emptyset\}$$
- Their membership function respectively read:
 
$$\mu_{\tilde{D}}(d) = \max(\mu_{Md^m(m)}, M^m(d)) \cdot (1 - \max(M(m)^F, M^m(d)))$$

$$\mu_{\tilde{D}}(d) = \min(\mu_{Md^m(m)}, \max(\max(M(m)^F, M^m(d)), \max(Md(m)^F, M^m(d))))$$
- We consider the prediction of the EE  $E = X < V_q$  with  $V_q$  the quantile of interest of the climatological distribution of  $X$ . The degree of consistency of  $E$  with the resulting possibility distribution and the degree to which the latter certainly implies it provide upper and lower bounds on the true probability of  $E$  [2]:
 
$$\max_{d \in \tilde{D}_{lower}} \mu_{\tilde{D}}(d) \leq P(X < V_q) \leq \max_{d \in \tilde{D}} \mu_{\tilde{D}}(d)$$

### 4. Test bed & Results

- We reproduce the experiment on an imperfect Lorenz 1996 model developed in [3]. The LP96 system was developed as a surrogate model for the atmospheric dynamics. The system dynamics is governed by the following coupled equations:
 
$$\frac{dX_1}{dt} = X_2(X_3 - X_1) - X_1 + P$$

$$\frac{dX_2}{dt} = -X_2(X_3 - X_1) - X_2 + Q$$

$$\frac{dX_3}{dt} = X_1(X_2 - X_3) + R$$
- $X_1$  is the variable of interest for prediction.  $X_i$  are randomly and independently drawn from  $N(X_i, 0, I^2)$ .
- We use a dataset of 2000 ensemble predictions associated to exact observations for the training of the parameter(s) of the membership function associated to  $M^m(m)$ : here, a unique (for exchangeable EMs) symmetrical triangular function.
- The objective function consists in minimizing the Brier score, here computed from the average of our probability interval:
 
$$Brier = \frac{1}{k} \sum_{i=1}^k (p_i - IV_i < V_q)^2$$
- We compare our results to those given by the direct model output (DMO) probabilistic prediction:
- Results for the quantile  $q=0.1$ . The Brier score is lower (mean and lower bound of our probability interval) than the DMO's. The approach is especially more interesting for the prediction of EE at small-medium lead times.
- Future works include to combine it to other rules (based on 'data analysis', to identify precursors to system behaviours) in order to get sharper bounds.

References [1] Coppi, D., et al. Proceedings of 19th IEEE Int'l Symposium on Intelligent Systems, 1994. [2] Williams, S. M., C. A. T. Ferson, and S. Ferson. QSRG. 140-149 (2014). [3] Dobson, D. and H. Prade. International Journal of Intelligent Systems 31.3 (2016): 215-229. [4] Wilks, Daniel S. Meteorological Applications 13.3 (2006): 243-254.