Incorporating expert opinion in an inferential model while retaining validity[1][2]

Ryan Martin
North Carolina State University
www4.stat.ncsu.edu/~rmartin
www.researchers.one

ISIPTA 2019
Ghent, Belgium
July 6th, 2019

[1] Joint work with my student, Mr. Leonardo Cella

[2] http://www.isipta2019.ugent.be/contributions/cella19.pdf

Introduction

- At a super-high level,
  - Bayesians require prior information
  - frequentists don't need/want it
- But most/all would agree that prior information *should* be used, whenever it's available.
- Key question: *How to incorporate prior information when it's "good" but not suffer from bias when it's not?*
- This paper is an attempt to answer this latter question.

Inferential models (IMs)

- Quick recap of the IM construction:
  - Associate data, parameters, and unobservable U
  - Predict unobserved U with a suitable random set, $\mathcal{S}$
  - Combine data, association, and random set to get belief and plausibility functions

$$\text{bel}_y(A) = \text{P}_{\mathcal{S}}\{\Theta_y(\mathcal{S}) \subseteq A\}$$
$$\text{pl}_y(A) = 1 - \text{bel}_y(A^c).$$

  - Use these for inference.
- *Good news:* "suitable" random set makes the inference *valid*...

IMs, cont.

Validity theorem.

Let $U \sim \mathsf{P}_U$ be as in the association, and choose a random set $\mathcal{S} \sim \mathsf{P}_{\mathcal{S}}$ on $\mathbb{U}$. Define $\gamma(u) = \mathsf{P}_{\mathcal{S}}(\mathcal{S} \ni u)$, $u \in \mathbb{U}$. If

$$\gamma(U) \geq_{\text{st}} \text{Unif}(0,1) \text{ when } U \sim \mathsf{P}_U,$$

then the corresponding inference on θ is *valid*, i.e.,

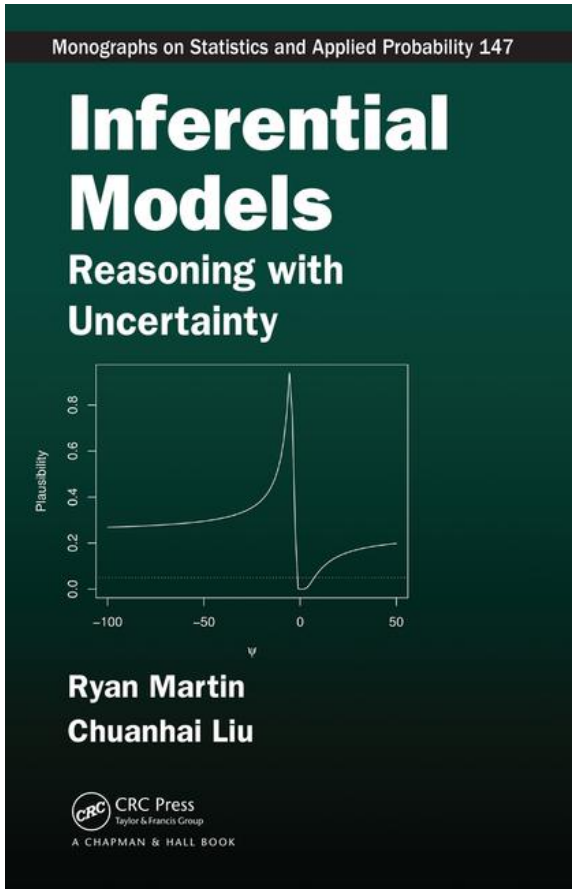
$$\sup_{\theta \notin A} \mathsf{P}_{Y|\theta}\{\text{bel}_Y(A) > 1-\alpha\} \leq \alpha, \quad \forall\, A, \quad \forall\, \alpha \in (0,1).$$

IMs, cont.

- *Bad news:* incorporating prior information, at least in the usual ways, messes up the desirable validity property...
- Leo and I were wondering whether it's possible to use the IM machinery in a nove way such that
  - prior information is incorporated
  - without sacrificing validity or efficiency.
- Our idea is based on *stretching* the random set.

Commercial

Wanna STRETCH the value of your money? You better act fast, the 20% off sale won't last long!



This paper

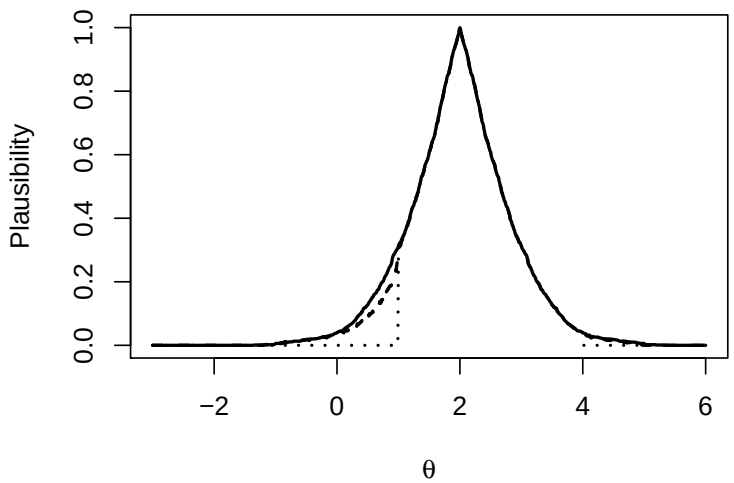
- Incorporating prior beliefs???
- Since I'm using belief functions, maybe I should combine IM output with prior beliefs via Dempster's rule.
- Unfortunately, Dempster's rule doesn't preserve validity.
- Our stretching idea goes roughly as follows:
  - calculate "agreement"[3] between data and prior information
  - if agreement measure is large
    - stretch the random set towards prior info
    - and shrink/contract in opposite direction
  - otherwise do nothing.
- If stretch/shrink step is done carefully,[4] then validity is preserved AND efficiency is never lost and sometimes gained.

[3] We make use of Dempster's rule here...

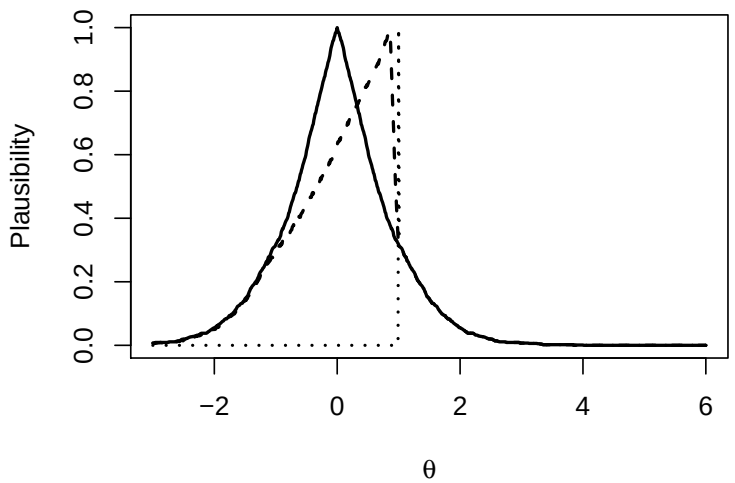
[4] Technically, key is to maintain the random set's "probability content"

Illustration

- Normal mean problem, $Y \sim \mathrm{N}(\theta, 1)$.
- Prior belief: "95% sure $\theta \in [1, 4]$"
- Plots of plausibility contour for $y \in \{0, 2\}$.
  - solid is based on IM only, no prior
  - dashed is based on stretch/shrink proposal



(a) $y = 2$



(b) $y = 0$

Illustration, cont.

- Simulation study, $Y \sim \mathrm{N}(\theta, 1)$.
- Prior belief: "95% sure $\theta \in B$"
- Compare 95% confidence intervals for θ
- Coverage probability for different true θ and B.

B	θ	Bayes	Dempster	IM_str
$[2, 9]$	3.0	0.930	0.974	0.953
	1.5	0.828	0.594	0.948
	0.0	0.701	0.809	0.956
	-4.0	0.239	0.955	0.941
$[2, 4]$	3.0	1.00	0.992	0.946
	1.5	0.080	0.601	0.957
	0.0	0.000	0.804	0.954
	-4.0	0.000	0.956	0.948

One more commercial

- The peer review system is still broken/detrimental.
- Our new platform (and non-profit org) offers researchers:
  - Autonomy to pursue their passions
  - Authority to develop and disseminate their work, and
  - Access to engage with the entire community of scholars

RESEARCHERS.ONE

- For details, check us out at

www.researchers.one

www.twitter.com/@ResearchersOne

The end

Thanks!

rgmarti3@ncsu.edu
www4.stat.ncsu.edu/~rmartin

RESEARCHERS.ONE