The Joy of Probabilistic Answer Set Programming

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Goal:

to show that the credal semantics for Probabilistic Answer Set Programming (PASP) leads to a very useful modeling language.

Answer set programming (ASP)...

- A program is a set of rules such as

$$\text{green}(X) \vee \text{green}(X) \vee \text{blue}(X) \quad :- \quad \text{node}(X), \textbf{not } \text{barred}(X)\textbf{.}$$

- A fact is a rule with no subgoals:

$$\text{node}(a)\textbf{.}.$$

Stable model semantics

- Herbrand base: all groundings generated by constants in the program.
- Minimal model is a model (interpretation that satisfies all rules) such that none of its subsets is a model.
- Answer set: a minimal model of the *reduct* (propositional program obtained by grounding, then removing rules with **not**, then removing negated subgoals).

Probabilistic ASP (PASP)

- A PASP program contains rules, facts, and *probabilistic facts*:

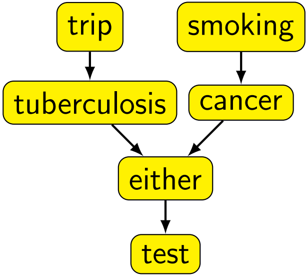
$$0.25 :: \text{edge}(\text{node1}, \text{node2}).$$

$$0.25 :: \text{edge}(\text{node2}, \text{node3}).$$

- A *total choice* induces an Answer Set Program.

Acyclic propositional (Bayesian network)

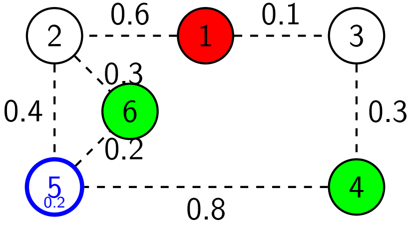
```
0.01 :: trip.
0.5 :: smoking.
tuberculosis :- trip, a1.
tuberculosis :- not trip, a2.
0.05 :: a1.        0.01 :: a2.
cancer :- smoking, a3.
cancer :- not smoking, a4.
0.1 :: a3.        0.01 :: a4.
either :- tuberculosis.
either :- cancer.
test :- either, a5.        0.98 :: a5.
test :- either, a6.        0.05 :: a6.
```



Stratified programs

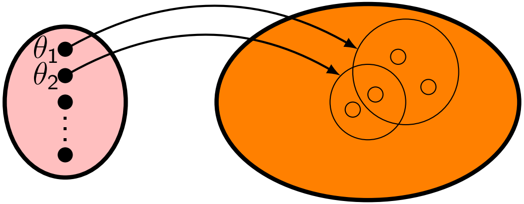
$\text{edge}(X, Y) :- \text{edge}(Y, X).$
$\text{path}(X, Y) :- \text{edge}(X, Y).$
$\text{path}(X, Y) :- \text{edge}(X, Z), \text{path}(Z, Y)..$

$0.6 :: \text{edge}(1, 2).$
$0.1 :: \text{edge}(1, 3).$
$0.4 :: \text{edge}(2, 5).$
$0.3 :: \text{edge}(2, 6).$
$0.3 :: \text{edge}(3, 4).$
$0.8 :: \text{edge}(4, 5).$
$0.2 :: \text{edge}(5, 6).$



PASP: Credal semantics

- A total choice may induce a program with many answer sets.



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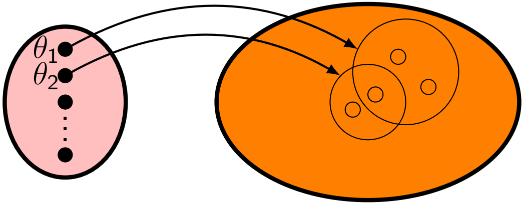
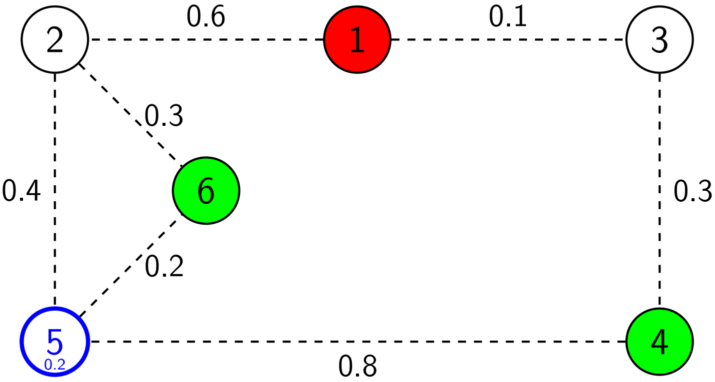


- Probability of each total choice may be distributed freely over answer sets: semantics is a credal set that dominates an infinitely-monotone capacity.

Is there a three-coloring?



Three-coloring

$$
\begin{aligned}
&\text{red}(X) \vee \text{green}(X) \vee \text{blue}(X) :- \text{node}(X).\\
&\text{edge}(X, Y) :- \text{edge}(Y, X).\\
&\neg\text{colorable} :- \text{edge}(X, Y), \text{red}(X), \text{red}(Y).\\
&\neg\text{colorable} :- \text{edge}(X, Y), \text{green}(X), \text{green}(Y).\\
&\neg\text{colorable} :- \text{edge}(X, Y), \text{blue}(X), \text{blue}(Y).\\
&\text{red}(X) :- \neg\text{colorable}, \text{node}(X), \textbf{not } \neg\text{red}(X).\\
&\text{green}(X) :- \neg\text{colorable}, \text{node}(X), \textbf{not } \neg\text{green}(X).\\
&\text{blue}(X) :- \neg\text{colorable}, \text{node}(X), \textbf{not } \neg\text{blue}(X)..
\end{aligned}
$$

Then: $\overline{\mathbb{P}}(\text{colorable}, \text{blue}(3)) = 0.976$.

Interpretation

- Lower/upper probabilities: *sharp* probabilities with respect to appropriate questions.
- "What is the probability that I will be able to select a three-ordering where node 2 is red?"
  - Answer is $\overline{\mathbb{P}}(\text{colorable}, \text{red}(2))$.

In the paper:

Algorithm to compute lower/upper probabilities!

Closing...

- In short: PASP with credal semantics is a very powerful language.
  - We can compute probabilities with some implicit quantification.