Semi-Graphoid Properties of Variants of Epistemic Independence based on Regular Conditioning

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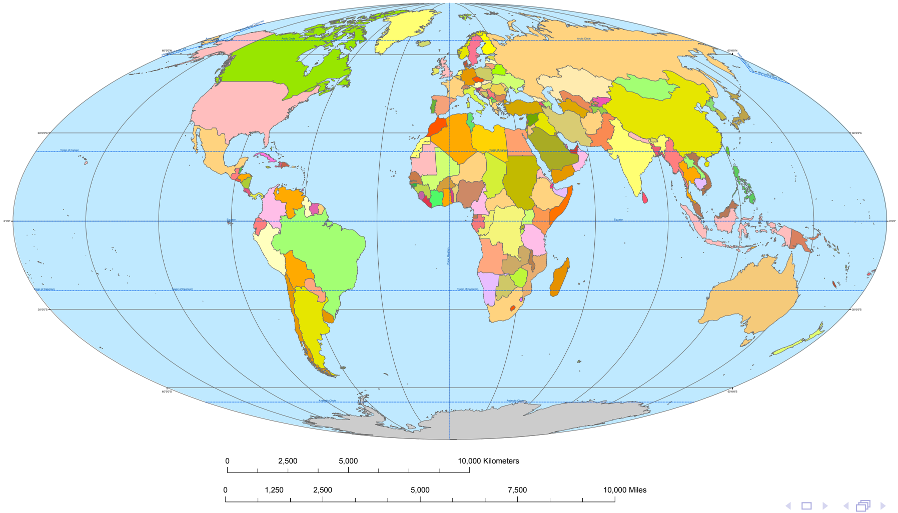

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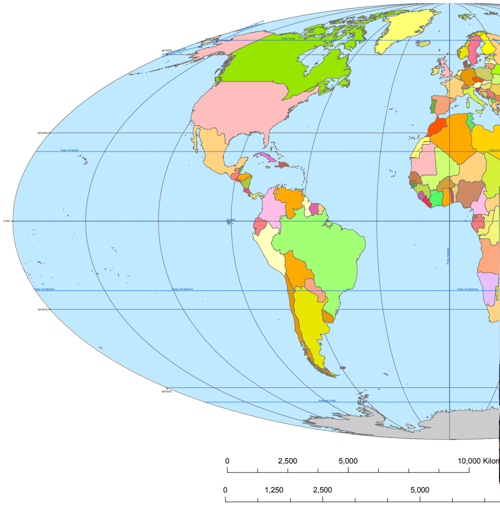

Goal:

to study the semi-graphoid properties of concepts of independence based on regular conditioning.

(With respect to credal sets that are general sets of Kolmogorovian-style probability measures.)

Semi-Graphoid Properties

Symmetry: $(X \perp\!\!\!\perp Y \mid Z) \Rightarrow (Y \perp\!\!\!\perp X \mid Z)$

Redundancy: $(X \perp\!\!\!\perp Y \mid X)$

Decomposition: $(X \perp\!\!\!\perp (W, Y) \mid Z) \Rightarrow (X \perp\!\!\!\perp Y \mid Z)$

Weak union: $(X \perp\!\!\!\perp (W, Y) \mid Z) \Rightarrow (X \perp\!\!\!\perp Y \mid (W, Z))$

Contraction: $(X \perp\!\!\!\perp Y \mid Z) \wedge (X \perp\!\!\!\perp W \mid (Y, Z)) \Rightarrow (X \perp\!\!\!\perp (W, Y) \mid Z)$

Regular conditioning:

$$\underline{\mathbb{E}}^{>}[f(X)|H] = \inf\{\mathbb{E}_{\mathbb{P}}[f(X)|H] : \mathbb{P} \in \mathbb{K}(X) \text{ and } \mathbb{P}(H) > 0\}$$

$$\text{whenever } \overline{\mathbb{P}}(H) > 0.$$

Irrelevance and independence

- Y is *regular-epistemically irrelevant* to X given Z:

$$\underline{\mathbb{E}}^{>}[f(X)|y,z] = \underline{\mathbb{E}}^{>}[f(X)|z] \qquad \text{whenever } \overline{\mathbb{P}}(y,z) > 0.$$

Symmetry fails: "symmetrize" to get corresponding independence.

Results

Theorem

If $(Y \text{ IR } X \mid Z)$ *denotes regular-epistemic irrelevance of* Y *to* X *given* Z*, then:*

- $(X \text{ IR } Y \mid X)$ *and* $(Y \text{ IR } X \mid X)$*;*
- *If* $(X \text{ IR } W, Y \mid Z)$*, then* $(X \text{ IR } Y \mid Z)$*;*
- *If* $(X \text{ IR } W, Y \mid Z)$*, then* $(X \text{ IR } Y \mid W, Z)$ *[NOTE: FAILS FOR de Finettian-conditioning!];*
- *If* $(Y \text{ IR } X \mid Z)$ *and* $(W \text{ IR } X \mid Y, Z)$*, then* $(W, Y \text{ IR } X \mid Z)$*.*

More results

Theorem

Regular-epistemic independence satisfies Symmetry and Redundancy.

Type-5 concepts

- Y is *type-5 epistemically irrelevant* to X given Z:

$$\underline{\mathbb{E}}^{>}[f(X)|B, z] = \underline{\mathbb{E}}^{>}[f(X)|z] \text{ whenever } \overline{\mathbb{P}}(B, z) > 0.$$

Symmetry fails: “symmetrize” to get corresponding independence.

Even more results

Theorem

If $(Y \text{ IR } X \mid Z)$ *denotes type-5 epistemic irrelevance of* Y *to* X *given* Z*, then:*

- $(X \text{ IR } Y \mid X)$ *and* $(Y \text{ IR } X \mid X)$*;*
- *If* $(X \text{ IR } W, Y \mid Z)$*, then* $(X \text{ IR } Y \mid Z)$*;*
- *If* $(X \text{ IR } W, Y \mid Z)$*, then* $(X \text{ IR } Y \mid W, Z)$*;*
- *If* $(W, Y \text{ IR } X \mid Z)$*, then* $(Y \text{ IR } X \mid Z)$*;*
- *If* $(W, Y \text{ IR } X \mid Z)$*, then* $(Y \text{ IR } X \mid W, Z)$*;*

And more results

Theorem

Type-5 independence and type-5 epistemic independence both satisfy Symmetry, Redundancy, Decomposition and Weak Union.

Also in the paper: complete and strong independence

- Complete independence satisfies all semi-graphoid properties.
- Strong independence satisfies Symmetry, Redundancy, Decomposition and Weak Union — but it fails Contraction!

Conclusion

- In this paper: a detailed map of semi-graphoid properties (all properties not mentioned fail...!).
- Confirmational/epistemic seem very weak... "type-5 condition" leads to better behavior.