Embedding Probabilities, Utilities and Decisions in a Generalization of Abstract Dialectical Frameworks

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Introduction

Decision Making

- Decision (action)
  - Uncertainties;
  - Preferences;

Introduction

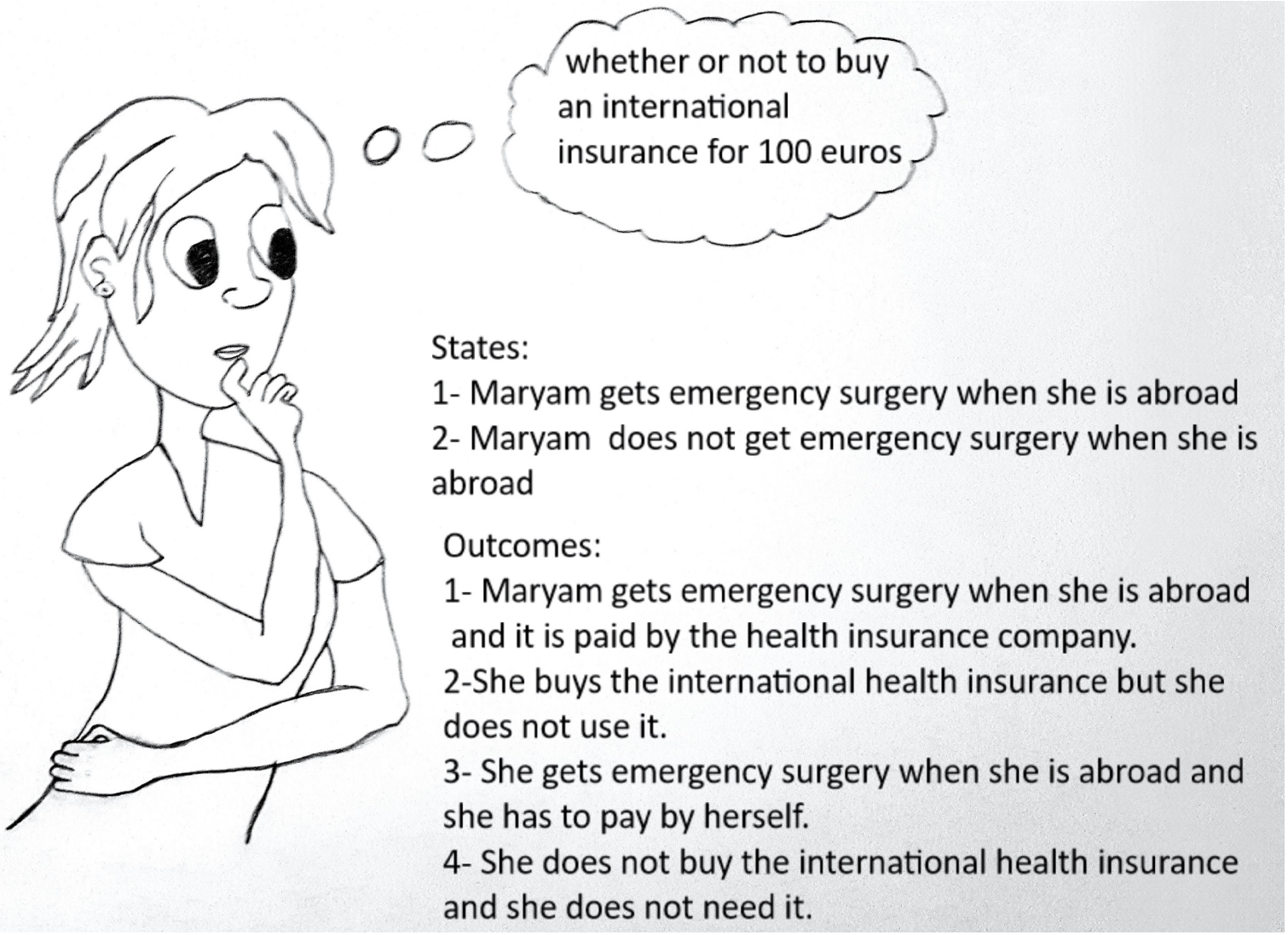
Decision Making

- Decision (action)
  - Uncertainties;
    - state
    - probabilities
  - Preferences;
    - outcome
    - utilities

Definition

Given $\succeq_p$ a rational order over the finite set of outcomes O. A function $u : O \to \mathbb{R}$ is called a **utility function** that represents $\succeq_p$ if, for every two outcomes o_1 and o_2, $u(o_1) \geq u(o_2)$ iff $o_1 \succeq_p o_2$.

Introduction



Decision Problem

Definition

A **decision problem** is a tuple (A, S, O, p, u) where:

- A is a finite set of actions;
- S is a finite set of states;
- O is a finite set of outcomes;
- p is a probability function on states, $p : S \to [0, 1]$ such that $\Sigma_s p(s) = 1$;
- u is a utility function on outcomes, $u : O \to [0, 1] \cap \mathbb{Q}$.

Decision Problem

Definition

Let (A, S, O, p, u) be a decision problem. The **expected utility** of $a \in A$ is defined as:

$$EU(a) = \Sigma_{o \in O} p(s|a, o)u(o)$$

- $p(s|a, o)$: probability of s combined with a, leads to o,
- $u(o)$: utility of o.

Definition

Maximum expected utility(MEU), $a \in$ MEU if for each $a' \in A$, $EU(a) \geq EU(a')$.

Motivation

- Formalisms, solvers and automated methods in decision theory
  - influence diagrams [Howard and Matheson, 2005, Olmsted, 1985, Shachter, 1986]

Question

Why are new approaches required?

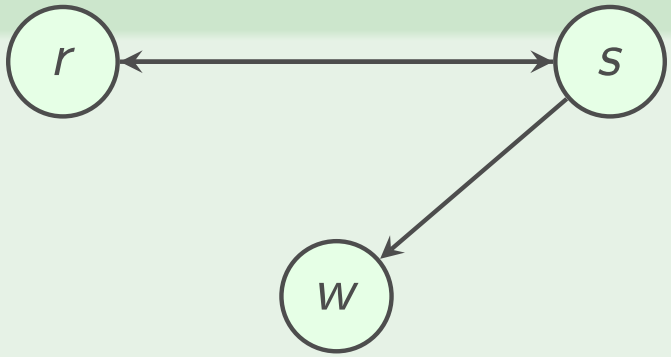
- Importance of decision making in human life
- wide variety of decision problems

Motivation

Argumentation formalism

- Abstract argumentation frameworks (AFs)[Dung, 1995]
- Abstract dialectical frameworks (ADFs) [Brewka and Woltran, 2010]
- Reasoning model: [Amgoud and Prade, 2009, Verheij, 2016]
- values, preferences [Vlek et al., 2016, Hunter and Thimm, 2014]

Example



Motivation

Argumentation formalism

- Abstract argumentation frameworks (AFs)[Dung, 1995]
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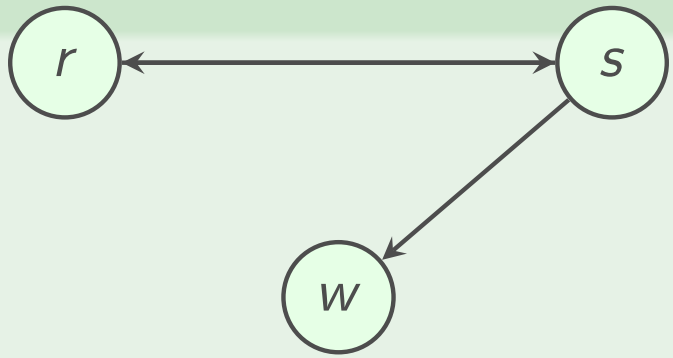


Question

Can an argumentation formalism be considered for modeling and solving decision problems?

Contributions

Main Contributions

- Introduce *numerical Abstract Dialectical Frameworks* (nADFs):
  - modeling decision problems,
  - solving the maximum expected utility of a problem.
- Choose the best action in the nADF of a decision problem.

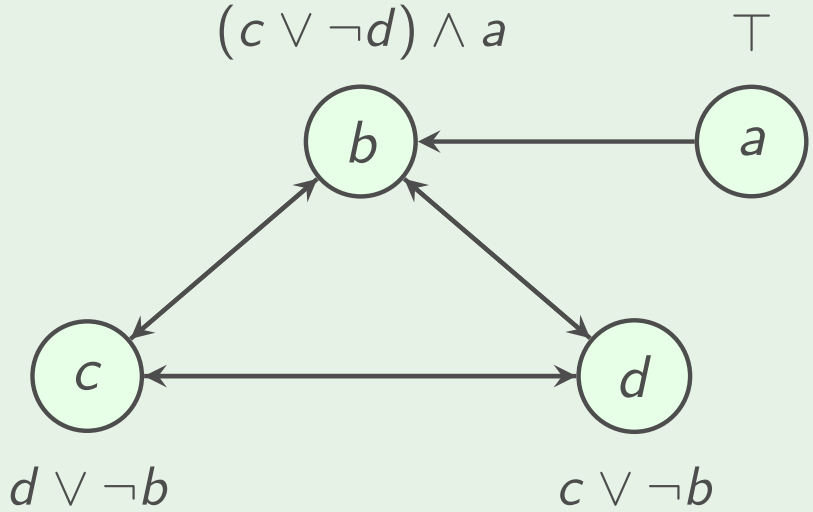
Background: ADFs

Definition

An abstract dialectical framework (ADF) is a tuple $D = (N, L, C)$ where:

- N is a finite set of nodes (arguments, statements, positions);
- $L \subseteq N \times N$ is a set of links;
- $C = \{\varphi_n\}_{n \in N}$ is a collection of propositional formulas $\varphi_n : (par(n) \to \{\mathbf{t}, \mathbf{f}\}) \to \{\mathbf{t}, \mathbf{f}\}$.

Example



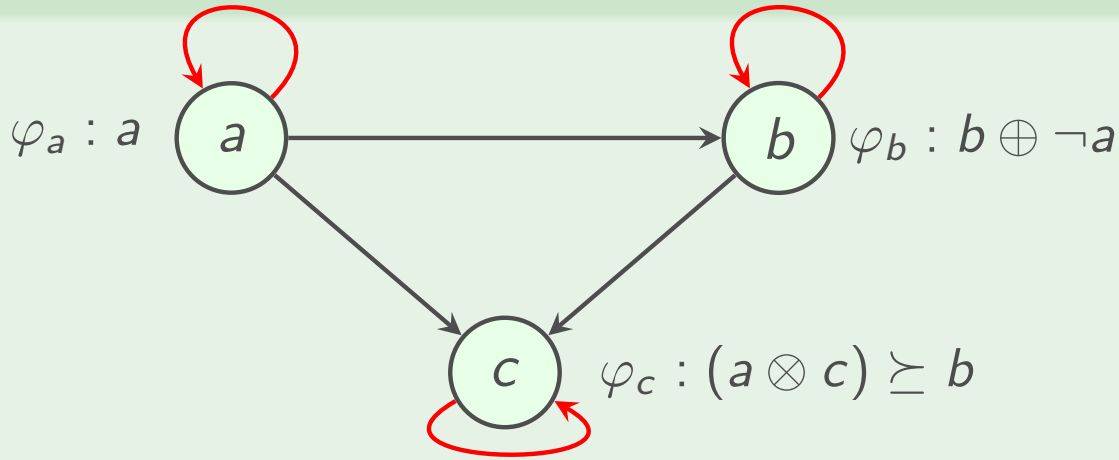
Numerical Abstract Dialectical Frameworks

Definition

Let V be $[0,1] \cap \mathbb{Q}$. An **nADF** is a tuple $U = (N, L, C, i)$

- N is a finite set of nodes;
- $L \subseteq N \times N$ is a set of links;
- $C = \{\varphi_n\}_{n \in N}$, $\varphi_n : (par(n) \to V) \to V$;
- i is an **input function** , $i : N' \to V$ where $N' \subseteq N$.

Example



Numerical Abstract Dialectical Frameworks

Information Ordering

- Many-valued interpretation: $v : N \to V_{\mathbf{u}},\ V_{\mathbf{u}} = ([0,1] \cap \mathbb{Q}) \cup \{\mathbf{u}\}$
- $\leq_i$: $\mathbf{u} \leq_i \mathbf{u}$ and $\mathbf{u} \leq_i x$, for $x \in ([0,1] \cap \mathbb{Q})$
- $w \in [v]_c$ iff $v \leq_i w$ and w is a total interpretation.

Characteristic Operator

$$\Gamma_U(v) : N \to V_{\mathbf{u}} \quad \text{with} \quad n \mapsto \bigsqcap \{w(\varphi_n) \mid w \in [v]_c\}.$$

Semantics of nADFs

- *admissible* in U iff $v \leq_i \Gamma_U(\mathbf{v})$;
- *complete* in U iff $v = \Gamma_U(\mathbf{v})$;
- *grounded* in U iff v is the $\leq_i$-least fixed point of Γ_U;
- *preferred* in U iff v is $\leq_i$-maximal admissible;
- *model* in U iff $v = \Gamma_U(\mathbf{v})$ and $\forall n \in N,\ v(n) \neq \mathbf{u}$;

Embedding of Decision Problems in nADFs

Definition

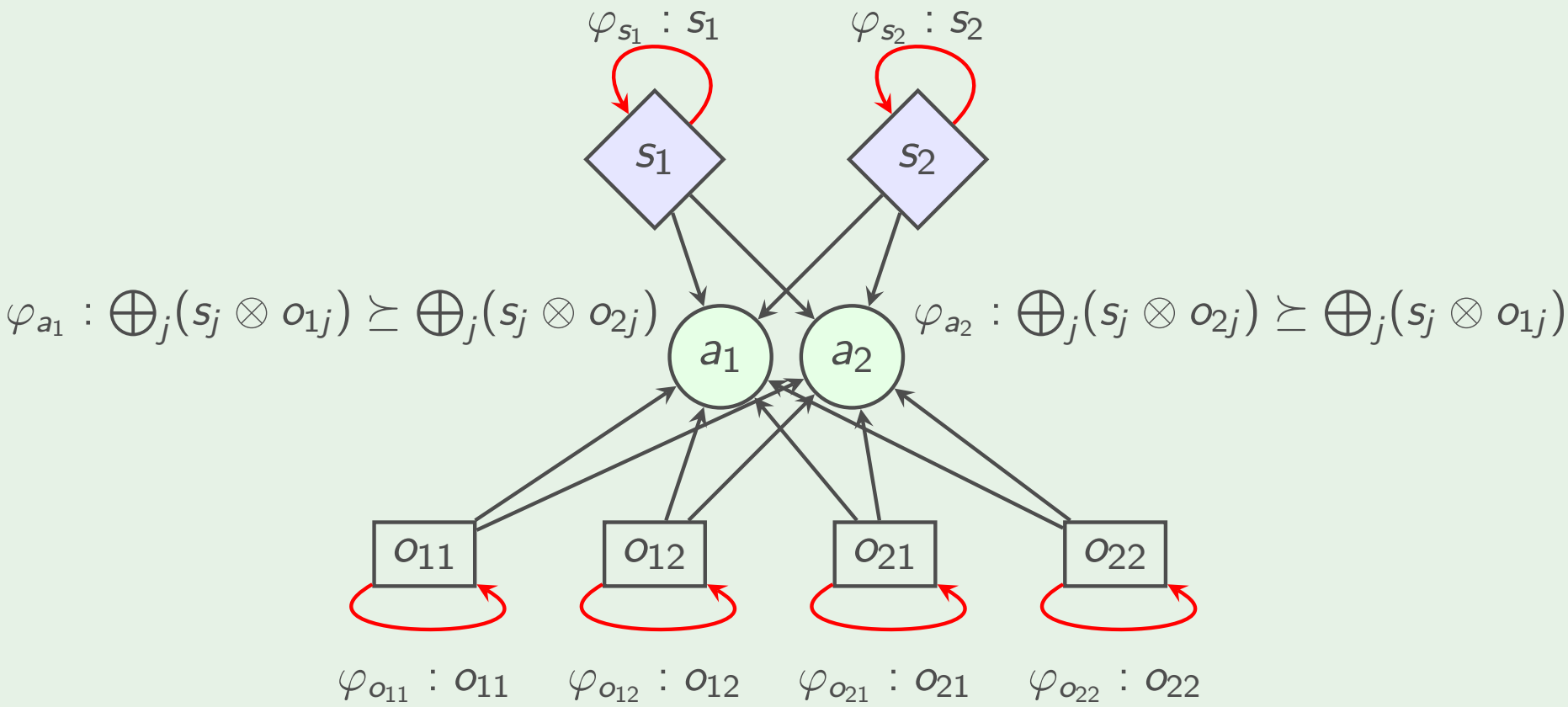
A decision problem $D = (A, S, O, p, u)$ can be modeled by nADF $U_D = (N, L, C, i)$ as follows:

- $N = A \cup S \cup O$;
- $\varphi_s = s$ for $s \in S$;
  $\varphi_o = o$ for $o \in O$;
  $\varphi_{a_i} = \bigotimes_{i \neq k}(\bigoplus_j(s_j \otimes o_{ij}) \succeq \bigoplus_j(s_j \otimes o_{kj}))$ for $a_i \in A$;
- $i(s) = p(s)$ for $s \in S$ and $i(o) = u(o)$ for $o \in O$.

Embedding of Decision Problems in nADFs

Maryam's decision problem as an nADF.

Example



Embedding of Decision Problems in nADFs

Theorem

Assume that a decision problem $D = (A, S, O, p, u)$ is modeled by nADF $U_D = (N, L, C, i)$. All semantics of U_D coincide.

Theorem

Let $D = (A, S, O, p, u)$ be a decision problem, let $U_D = (N, L, C, i)$ be the corresponding nADF, and let v be the grounded interpretation of U_D. The set A_1^v equals the set of actions with MEU in the decision problem D.

Related Works and Conclusion

Conclusion

- Argumentation is formally connected to decision making
- nADFs can model standard decision problems
- nADF that formalizes a decision problem, all semantics coincide
- nADF can be constructed for a decision problem to choose the best action

Future Work

- nADFs can be used for modeling decision problems in MAS
- nADFs are powerful enough to answer queries
- Computational complexity of decision problems in nADFs
- Experiments that show the effectiveness of nADFs

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Introduction

- Life is made up of long list of decisions, from choosing a healthy lunch to choosing a profession, affected by uncertainties and preferences.
  - The uncertainty mostly arises because of external factors, called **states**, out of control of agents; uncertainties can be modeled by probabilities.
  - An agent usually knows the set of possible **outcomes** of a decision and has a preferences on them; preferences can be modeled by utilities.
- **Expected Utility** deals with problems in which probabilities of states and utilities of outcomes play a role in the choice.
- **Argumentation theory** can shed light on the process of decision making, from modeling to evaluating a problem.
- Main goal is to propose an argumentation formalism, ***numerical abstract dialectical frameworks*** (nADFs) that can model decision problems.

Decisions and Argumentation

Decision Problem

- A **decision problem** is a tuple (A, S, O, p, u) where:
  - A is a finite set of actions;
  - S is a finite set of states;
  - O is a finite set of outcomes;
  - p is a probability function on states, $p : S \to [0, 1]$ s.t. $\Sigma_s p(s) = 1$;
  - u is a utility function on outcomes, $u : O \to [0, 1] \cap \mathbb{Q}$.
- The **expected utility** of $a \in A$ is defined as:
$$EU(a) = \Sigma_{o \in O} p(s|a, o)u(o)$$
- **Maximum expected utility**(MEU), $a \in$ MEU if for each $a' \in A$, $EU(a) \geq EU(a')$.

Argumentation Formalism

- An **abstract dialectical framework** (ADF) is a tuple $D = (N, L, C)$ where:
  - N is a finite set of nodes;
  - $L \subseteq N \times N$ is a set of links;
  - $C = \{C_n\}_{n \in N}$ is a collection of total functions $C_n : (par(n) \to \{\mathbf{t}, \mathbf{f}\}) \to \{\mathbf{t}, \mathbf{f}\}$.
- A **three-valued interpretation**: $v : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$.
- The **information ordering** $\leq_i$: $\mathbf{u} \leq_i \mathbf{t}$ and $\mathbf{u} \leq_i \mathbf{f}$.
- $v_i \leq_i v_j$ iff $\forall a \in A : v_i(a) \leq_i v_j(a)$.
- $[v]_c = \{w \in \mathcal{V}_c \mid v \leq_i w\}$
- $\Gamma_F(v)(n) = \prod\{C_n(w) \mid w \in [v]_c\}$
- Semantics of ADFs:
  - $v \in adm(F)$ if $v \leq_i \Gamma_F(v)$
  - $v \in pref(F)$ if v is $\leq_i$-maximal admissible
  - $v \in comp(F)$ if $v = \Gamma_F(v)$
  - v is $grd(F)$ if v is the $\leq_i$-least fixed point of $\Gamma_F(v)$
  - $v \in mod(F)$ if v is a two-valued interpretation and $v = \Gamma_F(v)$

Numerical Abstract Dialectical Frameworks

- nADFs enhance ADFs by allowing numerical acceptance conditions of arguments and arithmetical computations among them.
- The logic used in nADFs is a variation of propositional logic, consists of:
  - binary function symbols: $\oplus$ and $\otimes$;
  - a binary predicate symbol: $\succeq$
- Let V be $[0, 1] \cap \mathbb{Q}$. An **nADF** is a tuple $U = (N, L, C, i)$
  - N is a finite set of nodes;
  - $L \subseteq N \times N$ is a set of links;
  - $C = \{C_n\}_{n \in N}$, $C_n : (par(n) \to V) \to V$;
  - i is an input function , $i : N' \to V$ where $N' \subseteq N$.
- A **many-valued interpretation**: $v : N \to V_{\mathbf{u}}$, $V_{\mathbf{u}} = ([0, 1] \cap \mathbb{Q}) \cup \{\mathbf{u}\}$
- **i-correction** of v: $\mathbf{v}(n) = \begin{cases} i(n) & \text{if } i \text{ is defined on } n, \\ v(n) & \text{otherwise.} \end{cases}$
- The evaluation of non-standard connectives:
  - $\mathbf{v}(A \wedge B) := \min\{\mathbf{v}(A), \mathbf{v}(B)\}$
  - $\mathbf{v}(A \vee B) := \max\{\mathbf{v}(A), \mathbf{v}(B)\}$
  - $\mathbf{v}(a \otimes b) := \mathbf{v}(a) \times \mathbf{v}(b)$
  - $\mathbf{v}(a \oplus b) := \mathbf{v}(a) + \mathbf{v}(b)$
  - $\mathbf{v}(t_1 \succeq t_2) := \begin{cases} 1 & \text{if } \mathbf{v}(t_1), \mathbf{v}(t_2) \in \mathbb{Q} \text{ and } \mathbf{v}(t_1) \geq \mathbf{v}(t_2), \\ 0 & \text{if } \mathbf{v}(t_1), \mathbf{v}(t_2) \in \mathbb{Q} \text{ and } \mathbf{v}(t_1) < \mathbf{v}(t_2), \\ \mathbf{u} & \text{if either } \mathbf{v}(t_1) \text{ or } \mathbf{v}(t_2) \text{ is undecided.} \end{cases}$

Embedding of Decision Problems in nADFs

A decision problem $D = (A, S, O, p, u)$ can be modeled by nADF $U_D = (N, L, C, i)$ as follows:

- $N = A \cup S \cup O$;
- $\varphi_s = s$ for $s \in S$;
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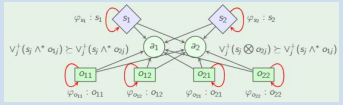
  $\varphi_{a_i} = \bigotimes_{i \neq k}(\bigoplus_j (s_j \otimes o_{ij}) \succeq \bigoplus_j (s_j \otimes o_{kj}))$ for $a_i \in A$;
- $i(s) = p(s)$ for $s \in S$ and $i(o) = u(o)$ for $o \in O$.

Example

a_1/a_2: whether or not to buy an international insurance for 100 euros.

- o_{11} buying and needing
- o_{12} buying and not needing
- o_{21} not buying but needing
- o_{22} not buying and not needing



Overview of results

Let $D = (A, S, O, p, u)$ be a decision problem, let $U_D = (N, L, C, i)$ be the corresponding nADF.
Theorem: All semantics of U_D coincide.
Theorem: Let v be the grounded interpretation of U_D.
The set A_1^v equals the set of actions with maximal expected utility in the decision problem D.

Thank you for your attention!

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Frame Title

Example