# Validity-preservation properties of rules for combining inferential models

Ryan Martin and Nicholas Syring
rgmarti3@ncsu.edu, and nasyring@wustl.edu

North Carolina State University and Washington University in St. Louis

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# Presentation Outline

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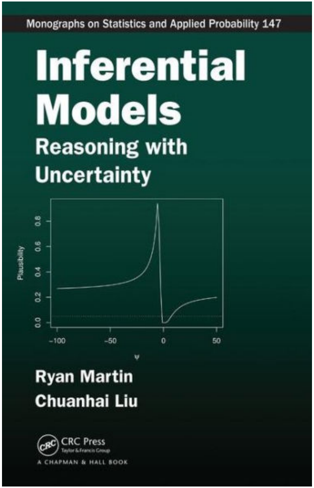
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# Presentation Outline

# Basic setup



A three step method to evaluate a statistical inference problem: associate, predict, and combine.

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Example: if $Y \sim \mathrm{N}(\theta, 1)$ and $\Phi$ denotes the standard normal CDF then

$$Y = \theta + U, \qquad U \sim \mathrm{N}(0, 1).$$

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Example: a sort of default random set is

$$\mathcal{S} = \{u : |u| < |U|, \; U \sim \mathrm{N}(0,1)\}.$$

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Let the solutions in $\vartheta$ for a given $(y, u)$ according to the association be denoted $\Theta_y(u) = \{\vartheta : y = a(\vartheta, u)\}$. Then, *combine* the solutions over $\mathcal{S}$ to obtain the random set on the parameter space

$$\Theta_y(\mathcal{S}) = \bigcup_{u \in \mathcal{S}} \Theta_y(u).$$

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Example: in the normal example this becomes the set

$$\{\vartheta : |y - \vartheta| < |U|\}, U \sim \mathrm{N}(0, 1).$$

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The inferential output for an assertion $A$ about $\theta$ is the belief/plausibility pair $(b_y(A), p_y(A))$ where

$$b_y(A) = \mathsf{P}_{\mathcal{S}}(\Theta_y(\mathcal{S}) \subseteq A), \quad \text{and} \quad p_y(A) = 1 - b_y(A^c).$$

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Example: for the normal example the plausibility contour function may be written

$$p_y(\{\vartheta\}) = 2(1 - \Phi(|y - \vartheta|))$$

# Presentation Outline

# Validity property

We insist the inferential model output is *valid*.

We rarely place high belief on false assertions.

*equivalently*

We rarely place low plausibility on true assertions.

# Validity property



Precisely,

$$\sup_{\theta \in A} \mathsf{P}_{Y|\theta}(p_Y(A) \leq \alpha) \leq \alpha$$

for every true $A$ and every $\alpha \in (0, 1)$.

By rare we mean calibrated to a uniform distribution.

# Valid inferential models



Our previous construction provides a valid inferential model whenever $\mathcal{S}$ is valid, details omitted. Not difficult to find a valid $\mathcal{S}$.

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How to do this?

1. Combine output? (the belief/plausibility functions)
2. Combine input? (the data)

# Possible strategies



1. Combining plausibility (contour) functions - via Dempster's Rule, (a version of) Dubois and Prade's Rule, perhaps others?

2. Combining data - using ideas from statistics like sufficiency, and a related PDE technique.

# See me at my poster!