A Short Note on the Equivalence of the Ontic and the Epistemic View on Data Imprecision for the Case of Stochastic Dominance for Interval-Valued Data

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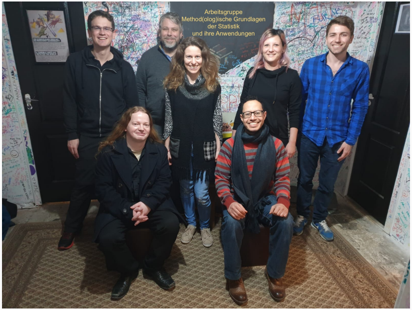

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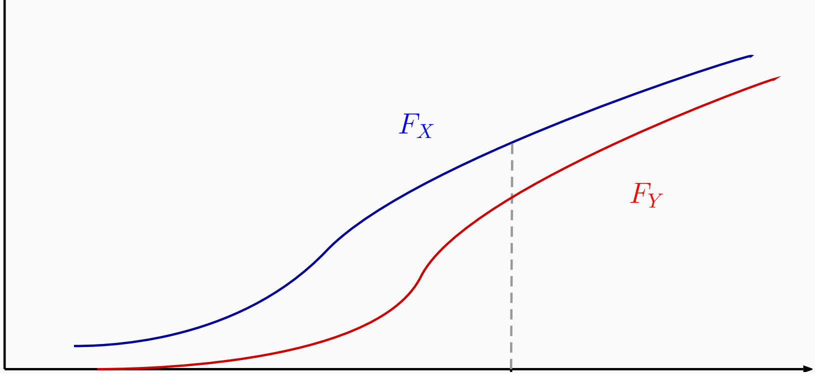
Ontic vs Epistemic Data Imprecision

- Epistemic view: A set-valued data point represents an imprecise observation of a precise, but not directlyobservable data point of interest.
- Ontic view: A set-valued data point is understood as a precise observation of somethinmg that is ’imprecise’ in nature.

First Order Stochastic Dominance 1a: univariate case

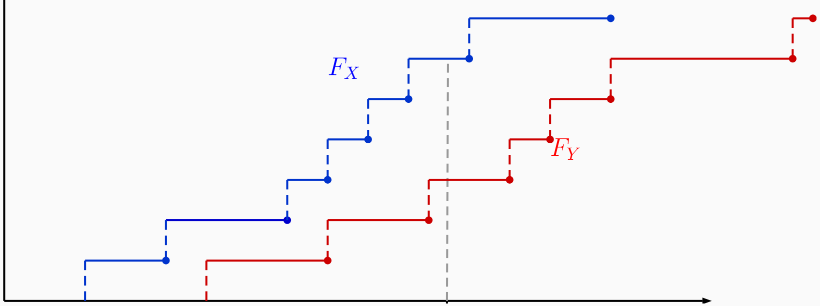
$$X \leq_{SD} Y \iff \forall c \in \mathbb{R} : F_X(c) \geq F_Y(c)$$
$$\iff \forall c \in \mathbb{R} : P(X \geq c) \leq P(Y \geq c)$$



First Order Stochastic Dominance 1b: univariate case, sample analogue

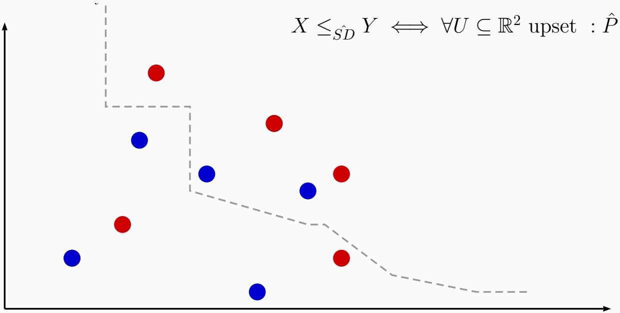
$$X \leq_{\hat{SD}} Y \iff \forall c \in \mathbb{R} : \hat{F}_X(c) \geq \hat{F}_Y(c)$$

$$\iff \forall c \in \mathbb{R} : \hat{P}(X \geq c) \leq \hat{P}(Y \geq c)$$



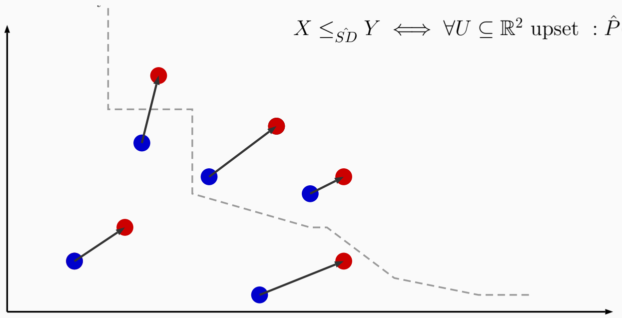
First Order Stochastic Dominance 2: bivariate case

$$X \leq_{\hat{SD}} Y \iff \forall U \subseteq \mathbb{R}^2 \text{ upset } : \hat{P}(X \in U) \leq \hat{P}(Y \in U)$$



(A set
$U \subseteq \mathbb{R}^2$ is called upset iff $\forall x \in U, y \in \mathbb{R}^2$ s.t. $y_i \geq x_i (i = 1, 2) \implies y \in U$)

First Order Stochastic Dominance 2: bivariate case



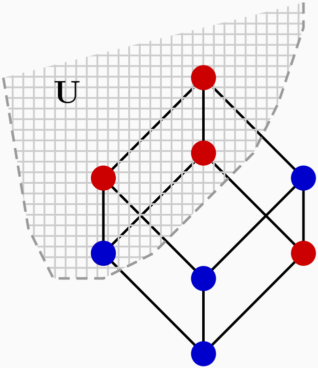
$$X \leq_{\hat{SD}} Y \iff \forall U \subseteq \mathbb{R}^2 \text{ upset } : \hat{P}(X \in U) \leq \hat{P}(Y \in U)$$

First Order Stochastic Dominance 3: general poset-valued case

- Given a poset $(V, \leq)$, a subset $U \subseteq V$ is called upset iff

$$x \in U \,, y \geq x \Longrightarrow y \in U.$$

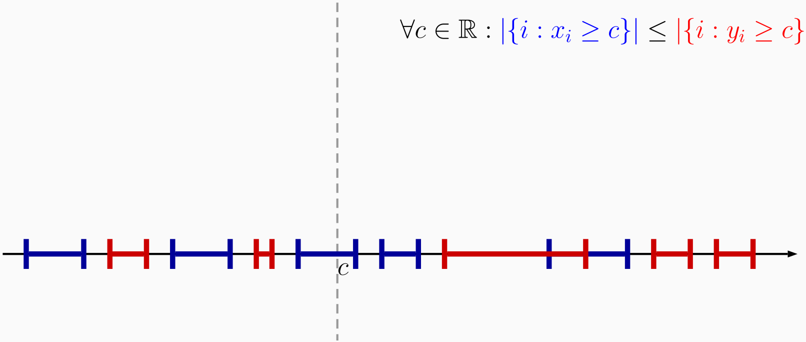
- $X \leq_{\hat{SD}} Y \iff \forall U \subseteq \mathbb{R}^2$ upset $: \hat{P}(X \in U) \leq \hat{P}(Y \in U)$



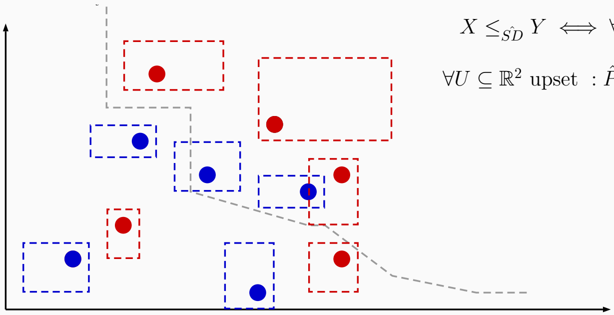
First Order Stochastic Dominance for Interval Data: univariate case, Epistemic view

$$X \leq_{\hat{SD}} Y \iff \forall x \in \mathfrak{x}, y \in \mathfrak{y}:$$

$$\forall c \in \mathbb{R} : |\{i : x_i \geq c\}| \leq |\{i : y_i \geq c\}|$$



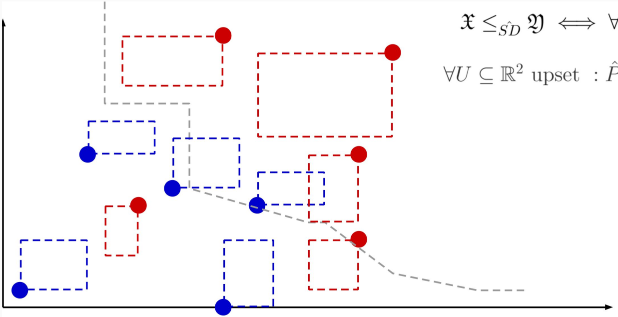
First Order Stochastic Dominance for Interval Data: bivariate case, Epistemic View



$$X \leq_{\hat{SD}} Y \iff \forall X \in \mathfrak{X}, Y \in \mathfrak{Y}:$$

$$\forall U \subseteq \mathbb{R}^2 \text{ upset } : \hat{P}(X \in U) \leq \hat{P}(Y \in U)$$

First Order Stochastic Dominance for Interval Data: bivariate case, Epistemic View



$$\mathfrak{X} \leq_{\hat{SD}} \mathfrak{Y} \iff \forall X \in \mathfrak{X}, Y \in \mathfrak{Y} :$$

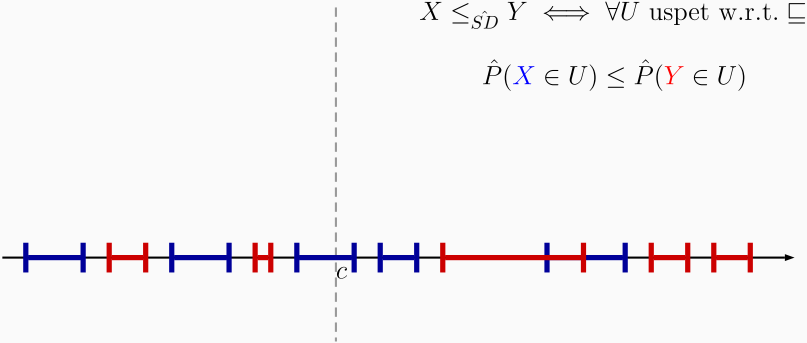
$$\forall U \subseteq \mathbb{R}^2 \text{ upset } : \hat{P}(X \in U) \leq \hat{P}(Y \in U)$$

First Order Stochastic Dominance for Interval Data: bivariate case, Ontic View

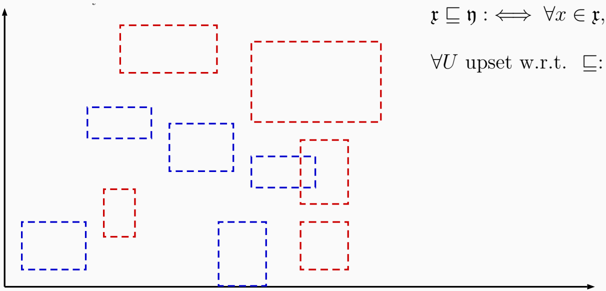
$$\mathfrak{x} \sqsubseteq \mathfrak{y} :\iff \forall x \in \mathfrak{x}, y \in \mathfrak{y} : x \leq y$$

$$X \leq_{\hat{SD}} Y \iff \forall U \text{ uspet w.r.t. } \sqsubseteq :$$

$$\hat{P}(X \in U) \leq \hat{P}(Y \in U)$$



First Order Stochastic Dominance for Interval Data: bivariate case, Ontic View



$$\mathfrak{x} \sqsubseteq \mathfrak{y} :\iff \forall x \in \mathfrak{x}, y \in \mathfrak{y} : x_i \leq y_i; i = 1, 2$$

$$\forall U \text{ upset w.r.t. } \sqsubseteq: \hat{P}(X \in U) \leq \hat{P}(Y \in U)$$

Simple Fact

Epistemic and Ontic view lead to the same results w.r.t. the presence of stochatsic dominance.