Two-State Imprecise Markov Chains for Statistical Modelling of Two-State Non-Markovian Processes

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Aim

present a new framework for

- fitting an imprecise two-state Markov chain
- to data generated by a two-state non-Markovian process
- via an imprecise version of MCMC

The Old Way

1. transition times $T_i \sim \mathrm{Exp}(\lambda_i)$
   (i.e. precise Markov chain assumed)
2. set of priors for λ_i
3. combine with data to get set of posteriors for λ_i
4. posterior predictive bounds on λ_i
5. use these bounds to fix an imprecise Markov chain

what is wrong with this approach?

The Old Way

what is wrong with this approach?

- model = set of distributions on parameters of a precise Markov chain
  imprecise Markov chains are not equivalent to this model
- sampling uncertainty ignored: only posterior predictive bounds are used
  no full uncertainty quantification
- imprecision does not reflect violations of stationarity & Markovianity
  wasted opportunity
- non-Bayesian analysis: inferences are not coherent with the model

The Newly Proposed Way

1. transition times $T_i \sim f_i(\lambda_i)$
   where f_i is a non-Markovian and/or non-stationary process
   and λ_i are unknown parameters of this process
2. set of prior distributions on λ_i
3. for each prior, use standard MCMC to sample posterior realizations of λ_i
   i.e. fit the non-Markovian process to the data
4. for each posterior sample of λ_i bound process by an imprecise Markov chain
5. produce sample of posterior bounds for any predictive quantity

The Newly Proposed Way

Nice features of this new approach:

- a form of imprecise MCMC: imprecision built into the predictive part
  this works because of Walley's marginal extension theorem
- fully coherent robust Bayesian approach:
  predictions are directly derived from posterior distribution
- imprecision reflects lack of data & non-Markovianity & non-stationarity
  (even if there is lot's of data, inferences can remain imprecise)

How Did We Do It

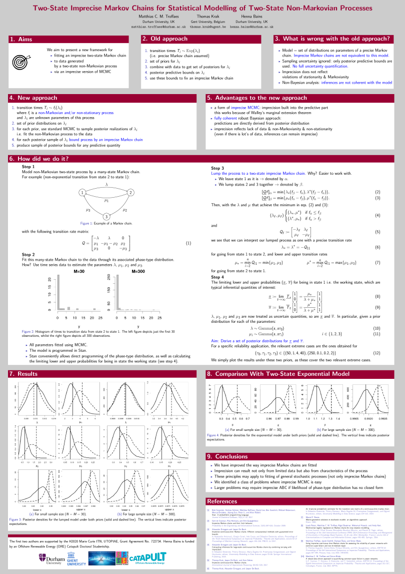
- non-Markovian behaviour modelled by a multi-state Markov chain
  can model any phase-type distribution for transition times!
  (every distribution can be approximated by a phase-type distribution)
- bounding by lumping multi-state Markov chains into imprecise two-state Markov chains
- details: see poster

Conclusions

- we've improved the way imprecise Markov chains are fitted
- imprecision results not only from limited data but also from characteristics of the process
- principles may apply to fitting of general stochastic processes
  (not only imprecise Markov chains)
- we identified a class of problems where imprecise MCMC is easy
- larger problems may require imprecise ABC
  if likelihood of phase-type distribution has no closed form

Thank You For Your Attention

We look forward to see you at our poster!



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